Reflecting on Reflections

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Target Grade: Geometry

Time Required: 75 minutes

Standards

Common Core Math Standards

- **G.CO.A.3** (IFD) Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry the shape onto itself.
- **G.CO.A.4** (IFD) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Lesson Objectives

Students will:

- Students can describe reflections of figures using multiple mathematical representations, including graphs, tables, and function notation.
- Students can identify lines of reflectional symmetry.

Central Focus

Reflections are all around in the real world. This lesson incorporates real world connections during the introductory activity. Students will see many images that are examples of real-life reflections. In addition, students will connect their knowledge of social media filters to the idea of mathematical reflections. This lesson uses the students' personal assets by relating the concept of reflection to familiar cultural trends. Many of the students like to watch Marvel movies and use social media platforms like TikTok. This app has a special filter that reflects the image on a screen over the vertical line running directly down the middle of the screen. This reflection is the same as reflecting figures that lie on the y-axis over the y-axis. The cultural connections will help engage students' attention as well as their knowledge of how reflections change objects.

Key Terms: reflection, reflectional symmetry, rigid transformations, line of reflection, symmetry

Background Information

This lesson plan has two following lesson plans:

- Lesson 2: All Turned Around -- Rotations
- Lesson 3: Introduction to Transformations

Visit the ORISE website to find the other two lesson plans.

Prior to this lesson, students will need to be familiar with graphing on the coordinate plane. Students must know how to plot points and read a graph on the coordinate plane. They must also use their knowledge of coordinate notation to describe functions algebraically. The students will engage in a discussion that acts as a refresher of the lesson from the day before, emphasizing the difference between rigid and non-rigid motion.

Prior to this lesson, teachers will need to be familiar with demonstrating reflections on a graph. The teacher should also be familiar with notations that relate to reflections as well as the following terms: reflection, reflectional symmetry, line of reflection, transformation, function, coordinate plane, symmetry, vertex, point, and line segment.

- Reflection
    - A reflection is a flip over a line specifically called the line of reflection.
    - Every point is the same distance from the line of reflection.
    - The reflection had the same shape as the original image. (Geometry - Reflection (mathsisfun.com))

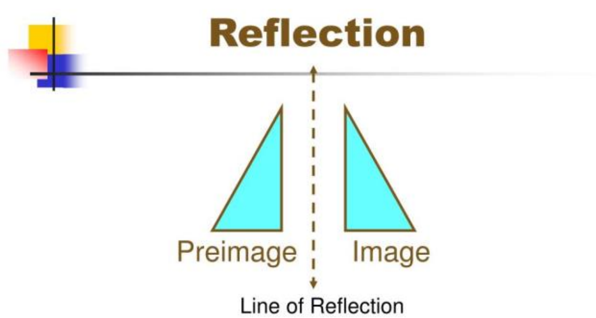

Figure 1:https://www.slideserve.com/gary-watkins/geometry

- Reflectional Symmetry
    - A type of symmetry which is with respect to reflections. (Reflection Symmetry - Definition, Shapes symmetry & Examples (byjus.com))

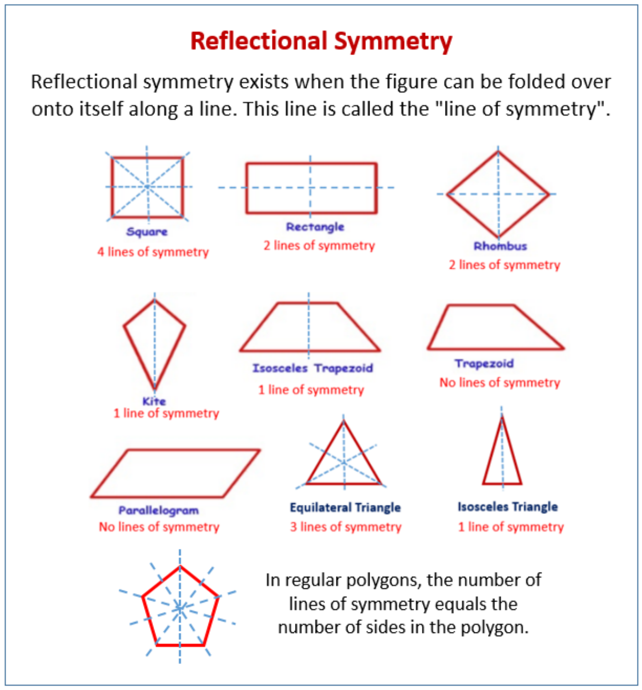

Figure 2: https://www.onlinemathlearning.com/transformation-symmetry.html

- Line of reflection
    - Across this line, a geometric shape or figure can be reflected across the line back to itself. (Reflection (math.net))

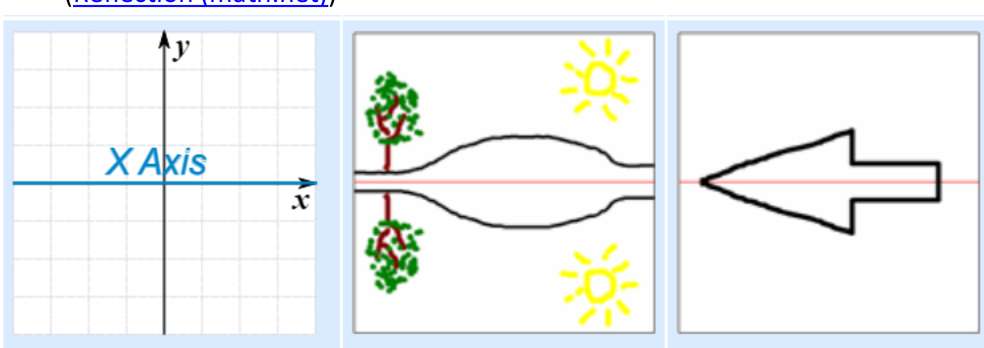

Figure 3: https://www.mathsisfun.com/geometry/symmetry-reflection.html

- Transformation
  - The three main types of transformations are rotation, reflection, and translation.
  - After any of the transformations, the shape still has the same size, area, angles, and line lengths.
    (Transformations (mathsisfun.com))

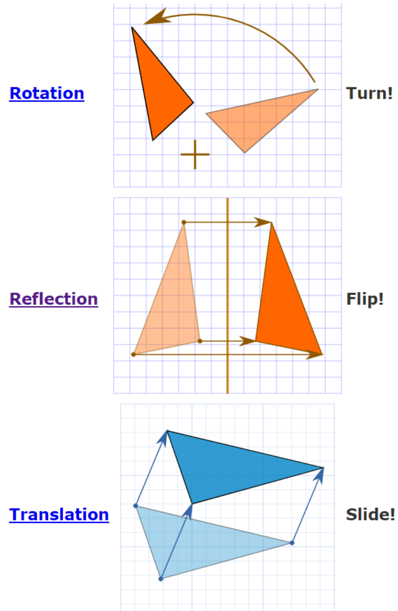

Figure 4: https://www.mathsisfun.com/geometry/transformations.html

- Function
  - A function relates an input to an output.
    (What is a Function (mathsisfun.com))

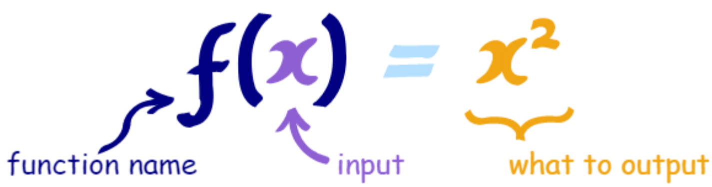

Figure 5: https://www.mathsisfun.com/sets/function.html

- Coordinate plane
    - The plane containing the “x” axis and “y” axis. (Definition of Coordinate Plane (mathsisfun.com))

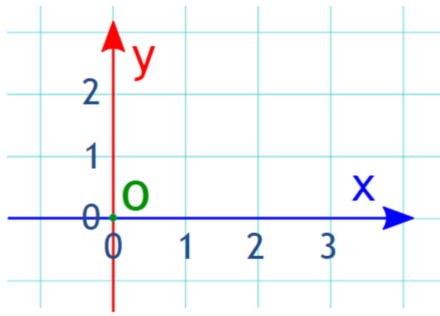

Figure 6: https://www.mathsisfun.com/definitions/coordinate-plane.html

- Symmetry
    - When two or more parts are identical after a flip, slide, or turn.
    - There are three main types of symmetry: reflectional symmetry, rotational symmetry, and point symmetry. (Symmetry Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

Figure 7: https://www.mathsisfun.com/definitions/symmetry.html

- Vertex
    - A point where two or more line segments meet. A corner. (Vertex Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

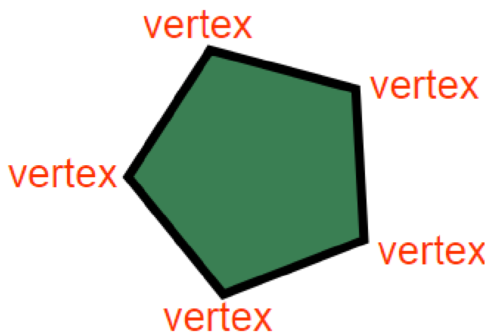

Figure 8: https://www.mathsisfun.com/definitions/vertex.html

- Point
    - A point is an exact location. It has no size, only position. (Point (mathsisfun.com))

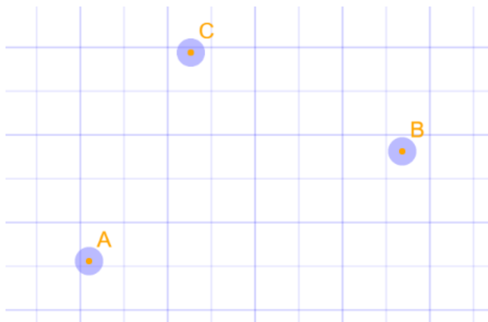

Figure 9: https://www.mathsisfun.com/geometry/point.html

- Line Segment
    - The part of a line that connects two points.
    - It is the shortest distance between two points.
    - It has a length. (Line Segment Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

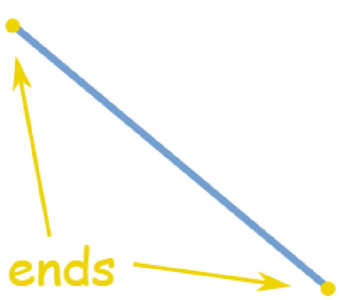

Figure 10: https://www.mathsisfun.com/definitions/line-segment.html

- Notations relating to reflection
    - A vs A' (pre-image vs. image

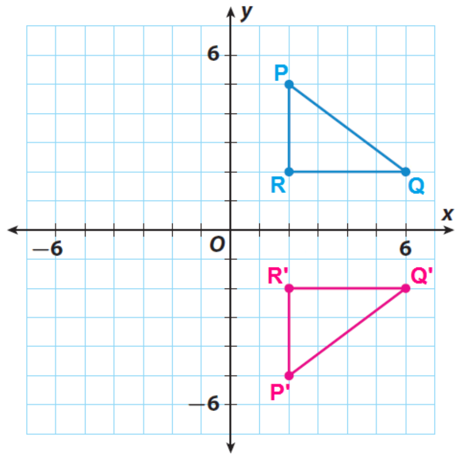

Figure 11: https://www.onlinemath4all.com/algebraic-representations-of-reflections.html

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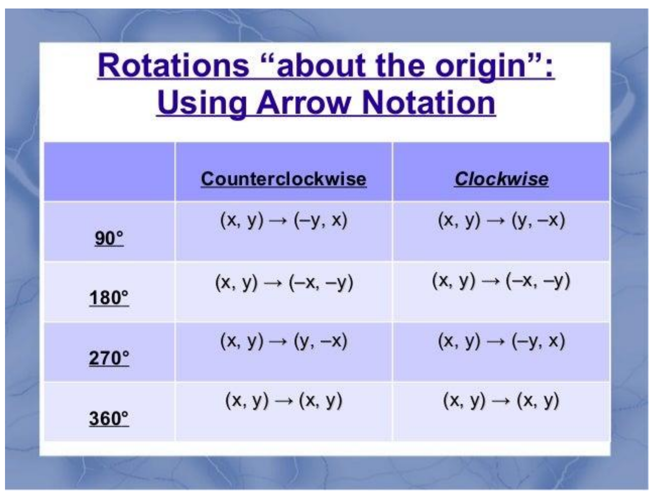
    - →



Rotations "about the origin": Using Arrow Notation

	Counterclockwise	Clockwise
90°	(x, y) → (−y, x)	(x, y) → (y, −x)
180°	(x, y) → (−x, −y)	(x, y) → (−x, −y)
270°	(x, y) → (y, −x)	(x, y) → (−y, x)
360°	(x, y) → (x, y)	(x, y) → (x, y)

Figure 12: https://www.slideshare.net/AcceleratedClasses/lesson10-transformational-geometry

Materials

- Reflections worksheet
- Reflections worksheet Key
- Reflections Exit Ticket
- Reflections Exit Ticket Key
- Patty paper
- "Reflections" Google Slides Presentation
- TV or projector
- Whiteboard

- Document Camera
- Transformation Coordinate Rules (Optional)
- Transformation Coordinate Rules Key (Optional)

Figure 13: Patty Paper
https://www.bing.com/images/search?view=detailV2&ccid=aCgTsL1G&id=CD6E17A22E2709707EAC606F4E75D7523B8887A5&thid=OIP.aCgTsL1G8TPUS3TrsQgx3gHaHZ&mediaurl=https%3a%2f%2fi5.walmartimages.com%2fasr%2f31adcaf7-5c3b-40b2-9059-29c4a21e4486.96b01615d10922bd4aaf

Instruction

Introduction (20 minutes)

Warmup (10 minutes)

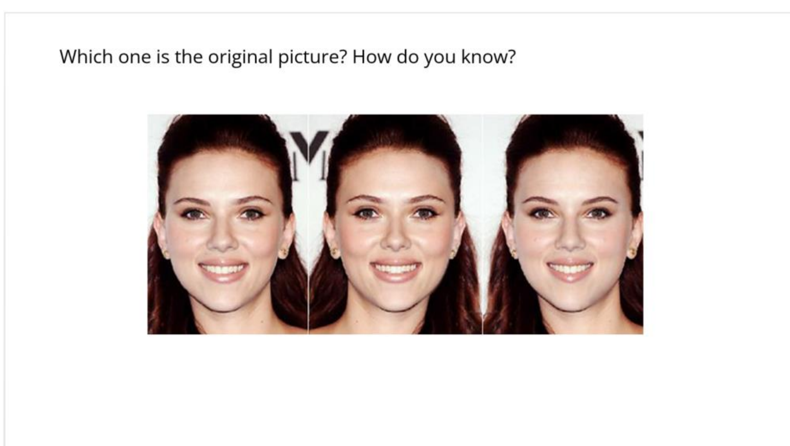
- On the board display the function (x, y) → (2x, 2y). The question underneath reads, “Is this function a rigid transformation? Why or why not?”
- The students have individual think time and time to confer with their neighbors before sharing their thoughts.
- As a whole class, have a discussion about the question above.
    - This discussion acts as a refresher of the lesson from the day before, emphasizing the difference between rigid and non-rigid motion.
    - The warm up activity activates prior learning and gives students a chance to use what they learned about algebraic function notation in a new context with a non-rigid motion.
    - Reviewing how to read this algebraic notation helps when the students describe reflections with this notation.
- Here are some possible questions to ask the students during the warmup:
    - Is this function a rigid transformation? Why or why not?
        - Here are some possible answers:
            - No, because the size of the figure is changing.
            - No, because it is using multiplication, specifically 2x and 2y.

- What does the word "rigid" mean?
  - Here are some possible answers:
    - Rigid means that the size does not change.
    - Rigid means that the figure moves, but the shape does not change.

Introduction to Activity (10 minutes)

- Using the presentation, show pictures of reflections in real-world pictures.

- Have a whole class discussion about the differences between the pre-images and images, highlighting lines of symmetry in the figures.
- On each slide, have the students answer the question of whether the transformation could also be a translation or not.
  - As a part of these real-life examples, have students look at images of Scarlett Johansson to identify the original image.
  - Then relate the modified images to reflections.



- As an extension, the students may talk about Snapchat and TikTok filters that use reflections to test face symmetry.
- Here are some possible questions to ask the students during the introduction:
    - Can you describe what is happening to the image of Scarlett Johansson?
    - How can we tell if an object is symmetrical?
        - Here are some possible answers:
            - Both sides are a reflection of each other.
            - When folded they line perfectly on top of each other.
    - Where is the line of symmetry?
        - Here are some possible answers:
            - The line where the image is being reflected.
            - The line where the image could be folded to have it line perfectly on top of each other.

Activity (35 minutes)

Part One (10 minutes)

- Pass around sheets of patty paper while the students view a picture of a figure on the graph displayed on the TV (the picture of the graph is below).
    - These activities make use of physical manipulatives to give students a way to concretely visualize the abstract movement of figures.

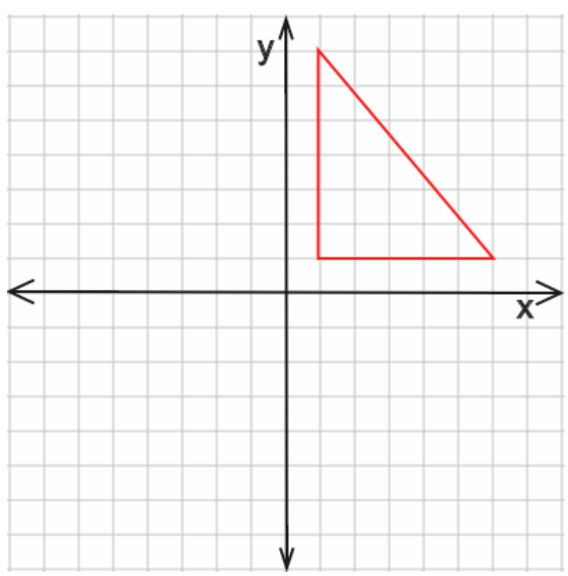
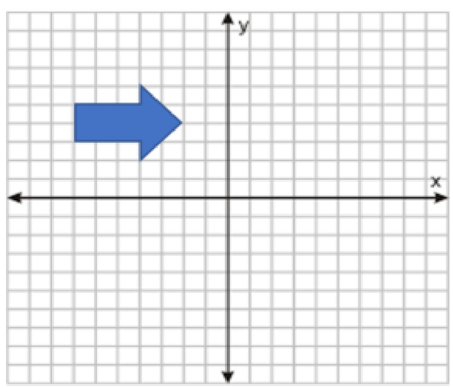


Figure 14: https://www.varsitytutors.com/ssat_middle_level_math-help/how-to-find-a-triangle-on-a-coordinate-plane

- Have the students do a Think-Pair-Share to describe how to reflect the image over the x-axis and how to represent the transformation with function notation algebraically.
  - The Think-Pair-Share allows students to increase their confidence in their thinking and engage in one-on-one discourse before engaging in discourse as a whole class.
- Call on a volunteer and a non-volunteer to describe their procedure and their algebraic representation.
- Then model how to use patty paper to draw the image of the reflection.
  - This activity also uses modeling to give students an idea of the correct way to do something before asking them to do it.
- As a class, discuss how to generalize a reflection over the x-axis as (x, y) → (x,-y).

Part Two (25 minutes)

- Hand out the reflections worksheet.
- Instruct students to work in groups of 3-4 on the reflections worksheet.
  - Have them use their patty paper to help them draw the images of the reflections.
  - Groups may work at different paces, and the faster groups may move on to problems 7-8 more quickly than their peers.



- During the group work, walk around the room to monitor the students and ask questions.
- As the teacher monitors the students, select the students' work to be shown during the discussion.
  - During the group work, select and sequence student work to be presented to show a range of solutions and ideas during the group discussion.
  - Sequencing student work helps guide logical thinking, connect mathematical representations, and help build student thinking from simple to complex.
- Once the majority of the groups have completed these problems, reconvene the class into a whole group to discuss their answers.
- Call on the students that were selected during the monitoring phase to present their solutions to the class using the document camera.

- Optional: Have the students fill out the Transformation Coordinate Rules.
    - This can help students in the future with how to do transformations on coordinate planes.
    - Note: There are other transformations on the page, so fill out the reflections section.
- Here are some questions to ask students while they work on their activity:
    - What would the image of this reflection look like?
    - What would it look like to flip this figure over the x-axis?
    - How did the x and y coordinates change? How can we describe that change using ordered pairs?
    - Can you use a table to find the pattern?
- Here are some questions to ask the students depending on the question they are working on:
    - 2. Does your image follow the function notation we came up with for a reflection over the x-axis?
    - 4. What would the shape look like if it was pointing down but had its tip at (4,10)?
    - 5. Draw the figure with the arrow pointing to the left or right. Can you find a line of reflection for that image?
    - 15. How do you know an object is symmetrical? How do reflections help us identify symmetry?
    - 17. Why would the isosceles triangle not have a horizontal line of symmetry? Which reflections could you perform to help you determine where the lines of symmetry are?

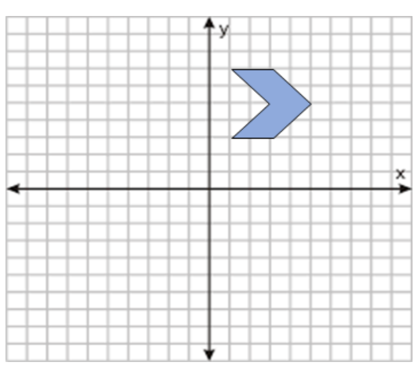
Conclusion (20 minutes)

Part 1 (10 minutes)

- Have a closing discussion that centers around the questions, “How do reflections relate to symmetry?” and “Which representations allow us to see symmetry in a figure?”

Part 2 (10 minutes)

- Instruct the students to complete the exit ticket, which requires them to draw the image of a reflection and describe a reflection that would map the image onto itself.
    - This allows the teacher to gauge student understanding to determine what the students know and what they still need to learn.



Differentiation

Students struggling with algebra:

- To help with the algebraic function notation, have a guide written on the board that says "(horizontal input, vertical input) → (horizontal output, vertical output)".
- The students benefit from peer support in group work.
- In addition, allow students to lean on the visuals during the lesson to help with their understanding.
- If they are having difficulty writing the algebraic notation, prompt them to make a table of points to compare the pre-image and the image of the figure and identify any patterns they see.
- Using the patty paper can help support the students by providing a physical procedure for drawing reflections.

Students who process information slowly:

- The think-pair-share helps students refine their thinking, and the group work allows them to be supported by their peers.
- The group work is chunked into two sections so that this student is not overwhelmed with information.
    - Everyone is expected to do the first section (problems 1-4), and some students may complete the second section (problems 5-8).
- During the group work, tell the group to read the instructions for each problem out loud and discuss what they need to do before starting to work.

Students who need an extra challenge:

- Allow them to work at an accelerated pace during the group work.
- If they finish the first section early, they may move on to the next page early and present these problems to the class.

Grouping:

- Peer interaction acts as a means of differentiation. The heterogeneous groups allow for more advanced students to solidify their understanding as they support their peers, and students who need extra support have guidance from their group members.
- Group students by the arrangement of their desks into groups of 3-4 students. Have groups be mixed ability grouping for peer support and collaborative learning.

Assessment

Formative Assessment

- The warmup shows the teacher if students can connect the algebraic representation of a transformation to how the transformation changes an object graphically.
- Discussions allow the teacher to check students thinking as they learn and correct any misconceptions as they occur.
- The teacher can monitor group work and ask assessing questions to check for understanding as students practice their skills and apply their knowledge.
    - During group work, the teacher can see if students can use patty paper correctly, plot images accurately, and identify symmetry in a figure.
    - The information gained from this formative assessment helps guide the whole class discussion to center on what the students misunderstood and what they found challenging.
- The closure makes use of formative assessment that allows the teacher to gauge student understanding to determine what the students know and what they still need to learn.

Summative Assessment

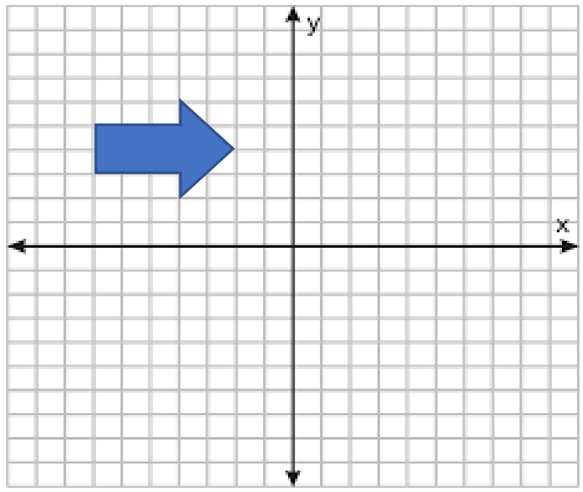
- The exit ticket shows the teacher if students can read an algebraic description of a reflection and connect it to the graphical representation of the reflection. It also shows the teacher if students understand symmetry and its relation to reflections.

Reflections

Why is this not a true mathematical reflection?

Which one is the original picture? How do you know?

Reflections Exploration



1) Predict the direction of the arrow after a reflection…

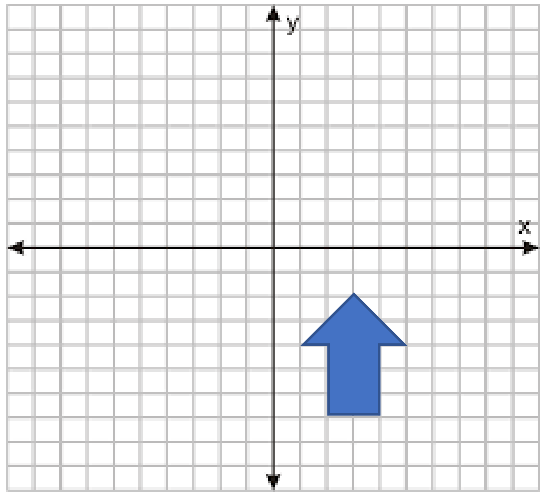
a. across the y-axis

b. across the x-axis

2) Draw the reflections from 1. Were you correct?

3) Describe a reflection that would map the arrow so that it is pointing to the left with its tip at (-2, 4).

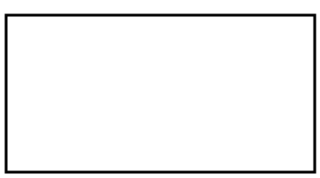
4) What reflection could map the arrow so that it is pointing up or down?

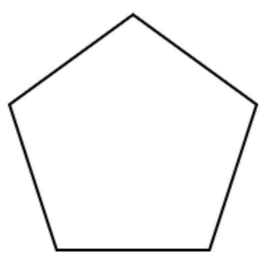


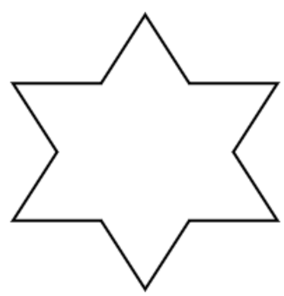
5) Draw the reflection described by the arrow notation (x, y) → (x, -2 – y)

6) Identify the line of reflection.

7) If there is a line of reflection that will map a figure onto itself, then that figure has <u>reflectional symmetry</u>. This line is the line of symmetry. For each figure, draw the lines of



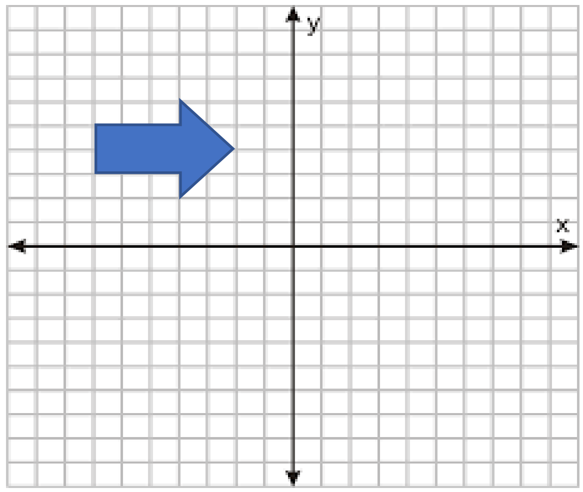




symmetry.

8) How many lines of symmetry does a circle have?

Reflections Exploration KEY



1) Predict the direction of the arrow after a reflection…

a. across the y-axis

b. across the x-axis

Answers may vary.

2) Draw the reflections from 1. Were you correct?

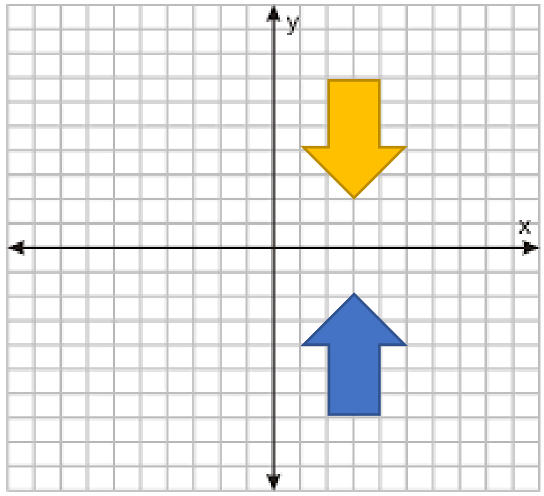
Answers may vary.

3) Describe a reflection that would map the arrow so that it is pointing to the left with its tip at (-2, 4).

Answers may vary.

4) What reflection could map the arrow so that it is pointing up or down?

Answers may vary.



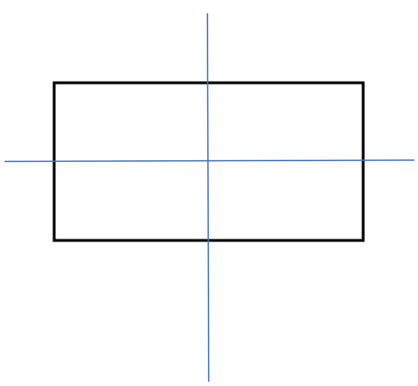
5) Draw the reflection described by the arrow notation (x, y) → (x, -2 – y)

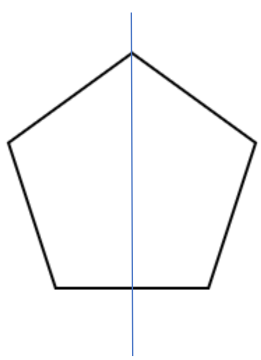
ANSWER shown with yellow arrow.

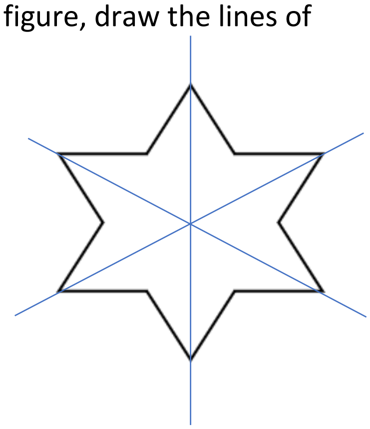
6) Identify the line of reflection.

x-axis or y = 0

7) If there is a line of reflection that will map a figure onto itself, then that figure has <u>reflectional symmetry</u>. This line is the line of symmetry. For each figure, draw the lines of







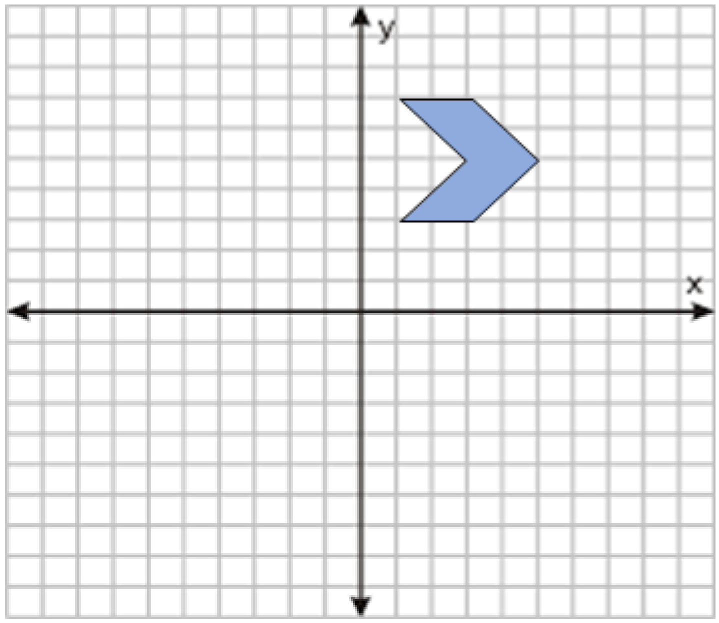
symmetry.

8) How many lines of symmetry does a circle have?

infinite

Exit Ticket

Name: ___________________________



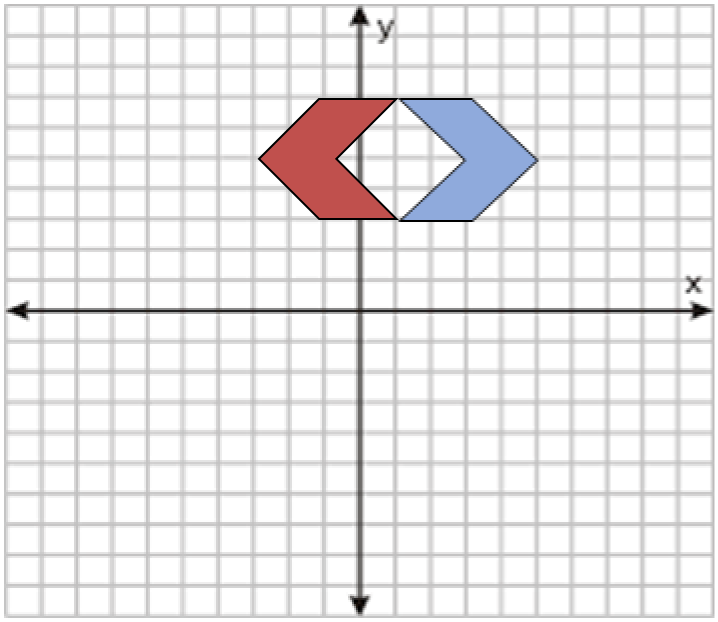
1. Draw the reflection of the arrow described by the function (x, y) → (2 - x, y).

2. Identify the line of reflection.

3. Describe a reflection that would map the arrow onto itself.

Exit Ticket

Name: KEY



1. Draw the reflection of the arrow described by the function (x, y) → (2 - x, y).

2. Identify the line of reflection.

   x = 1

3. Describe a reflection that would map the arrow onto itself.

   Answers may vary.

Transformation Coordinate Rules

Translations

Reflections

Over the x-axis:

Over the y-axis:

Over the line y = x:

Over the line y = -x:

Rotations

Center of rotation (0,0)

90˚ counterclockwise or ____ clockwise turn:

180˚ counterclockwise or ____ clockwise turn:

270˚ counterclockwise or ____ clockwise turn:

Transformation Coordinate Rules KEY

Translations

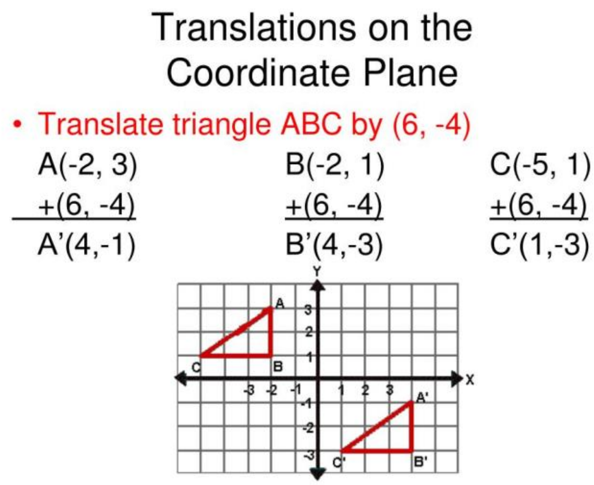


Figure 1: https://www.slideserve.com/theola/translations-on-the-coordinate-plane

Reflections

Over the x-axis: $(x, y) \rightarrow (x, -y)$

Over the y-axis: $(x, y) \rightarrow (-x, y)$

Over the line y = x: $(x, y) \rightarrow (y, x)$

Over the line y = -x: $(x, y) \rightarrow (-y, -x)$

Rotations

Center of rotation (0,0)

90° counterclockwise or 270 degrees clockwise turn:

$(x,y) \rightarrow (-y, x)$

180° counterclockwise or 180 degrees clockwise turn:

$(x,y) \rightarrow (-x, -y)$

270° counterclockwise or 90 degrees clockwise turn:

$(x,y) \rightarrow (y, -x)$