Rotations- All Turned Around

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Target Grade: Geometry

Time Required: 75 minutes

Standards

- **G.CO.A.2** (IFD) Represent transformations in the plane in multiple ways, including technology. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).
- **G.CO.A.3** (IFD) Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry the shape onto itself.
- **G.CO.A.4** (IFD) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Lesson Objectives

Students will:

- Describe rotations of a figure using writing, graphs, tables, and arrow notation.
- Identify and describe rotational symmetry in a figure.

Central Focus

Rotations are all around us in the real world. Car wheels and bike wheels, gears, and Ferris wheels all rotate. During class, the students will analyze the rotation of the hands of a clock and apply the academic language involved in measuring degrees to describe the rotation of clock hands.

Key Terms: rotation, center of rotation, rotational symmetry, function, pre-image, image, point, vertex, angle

Background Information

This lesson is the second part to a three part lesson. Look for the following on the ORISE website:

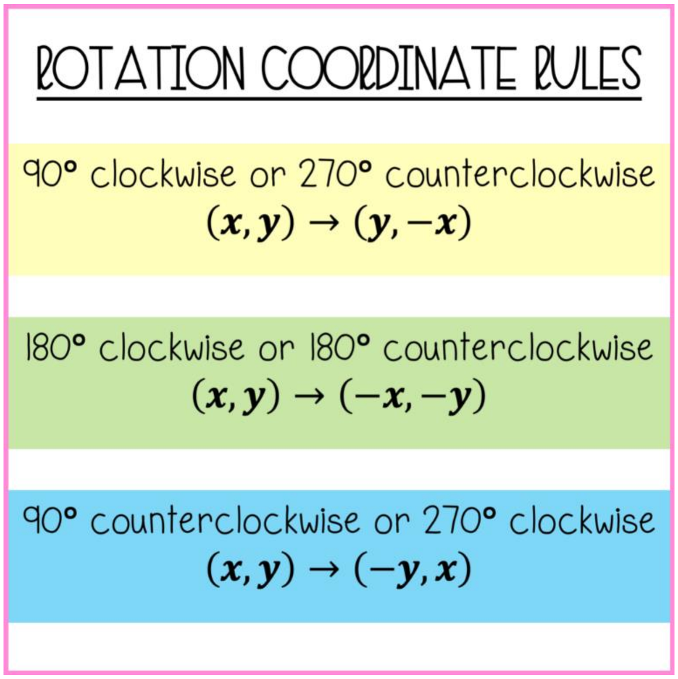
- Lesson 1: Reflecting on Reflections
- Lesson 3: Introduction for Transformations

Rotations are the last type of rigid transformation the students will learn about. Students will use their knowledge of angles and angle measures to complete this lesson. The function notation for rotations is similar to function notation for the other rigid transformations. In addition, students will build off their understanding of symmetry and apply their knowledge of reflectional symmetry to rotations. Like the lessons on translations and reflections, this lesson uses the students' prior knowledge of graphing on the coordinate plane, which includes reading graphs and plotting points. It also uses their knowledge of angle measures to describe and draw rotations given a specific angle of rotation. Students will apply their learning of algebraic representations of transformations in the previous 2 lessons to describe rotations with arrow notation. Additionally, this lesson also builds off the students' knowledge of symmetry within a figure. They must apply the idea of symmetry that they used for reflections in the previous lesson to rotations in this lesson. In particular, they will use their knowledge of what it means when you "map a figure onto itself."

Prior to this lesson, teachers should be familiar with clockwise and counter clockwise rotations and how they relate to rotational symmetry. Teachers should also be familiar with how to rotate a figure on a coordinate plane. Teachers should be familiar with the coordinate rules, but may also show the students how to rotate a figure on a coordinate plane without using the coordinate rules. Video links describingboth methods are below. Teachers should also be familiar with the terms: rotation, center of rotation, rotational symmetry, function, pre-image, image, point, vertex, and angle.

- Here is a video about rotations on the coordinate plane using coordinate rules:
  - https://www.youtube.com/watch?v=44W8AQZ4at8
- Here is a video about rotations on the coordinate plane without using coordinate rules:
  - https://www.youtube.com/watch?v=7vKxhfPMyAo

- Here are the list of coordinate rules in a table format:



ROTATION COORDINATE RULES

90° clockwise or 270° counterclockwise $(x, y) \rightarrow (y, -x)$
180° clockwise or 180° counterclockwise $(x, y) \rightarrow (-x, -y)$
90° counterclockwise or 270° clockwise $(x, y) \rightarrow (-y, x)$

Figure 1: https://lindsaybowden.com/3-ways-to-rotate-a-shape/

- Rotation
  - A circular movement.
  - Rotation has a central point that stays fixed and everything else moves around that point in a circle.
  - A "Full Rotation" is 360 degrees. ([Rotation Definition (Illustrated Mathematics Dictionary) (mathsisfun.com)](https://www.mathsisfun.com/definitions/rotation.html))

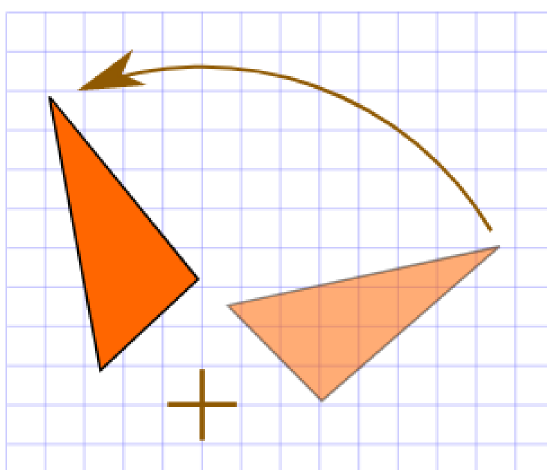


Figure 2: https://www.mathsisfun.com/definitions/rotation.html

- Center of rotation
    - The point at which the figure rotates around.
    - In the image below the center rotation is marked with a "+."

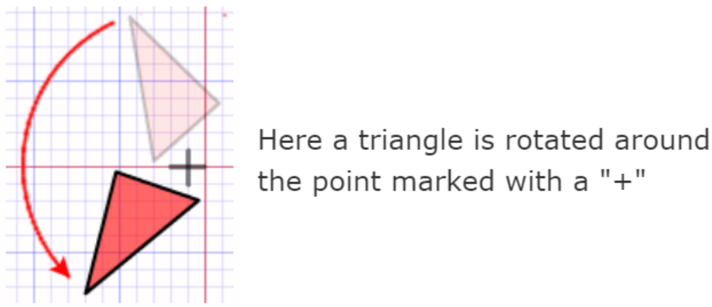


Figure 3: https://www.mathsisfun.com/geometry/rotation.html

- Rotational Symmetry
    - A shape has rotational symmetry when it still looks the same after some rotation (of less than one full turn).
    (Rotational Symmetry (mathsisfun.com))

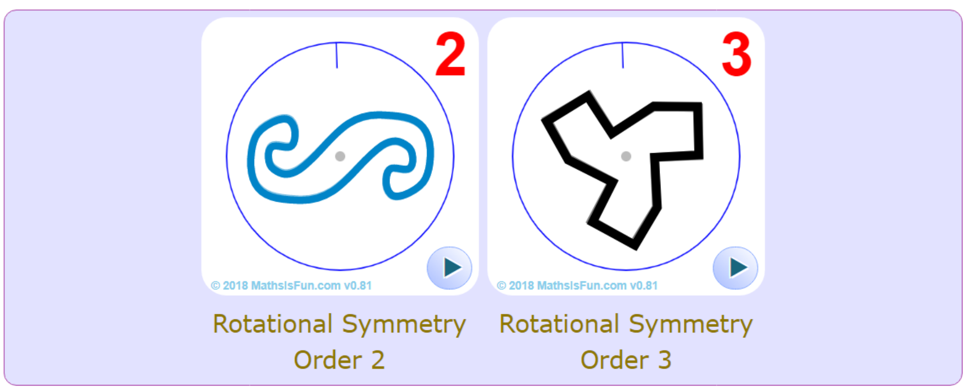


Figure 4: https://www.mathsisfun.com/geometry/symmetry-rotational.html

- Function
    - A function relates an input to an output.
    (What is a Function (mathsisfun.com))

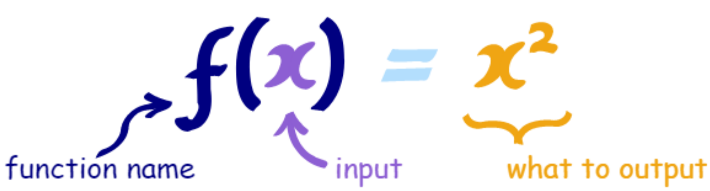


Figure 5: https://www.mathsisfun.com/sets/function.html

- Pre-image
    - The original, unaltered shapes are called preimages. ([Preimage & Image - Calculus How To]())

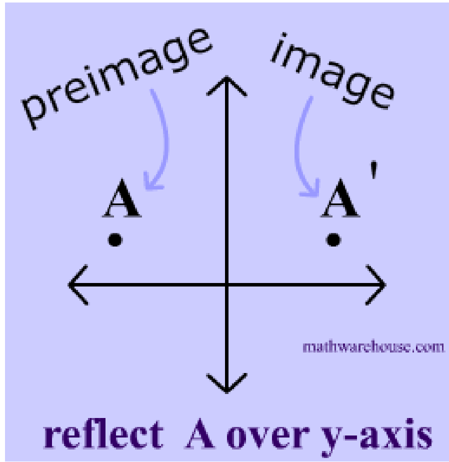


Figure 6: https://www.mathwarehouse.com/transformations/

- Image
    - The new (transformed) shapes are called images. ([Preimage & Image - Calculus How To]())

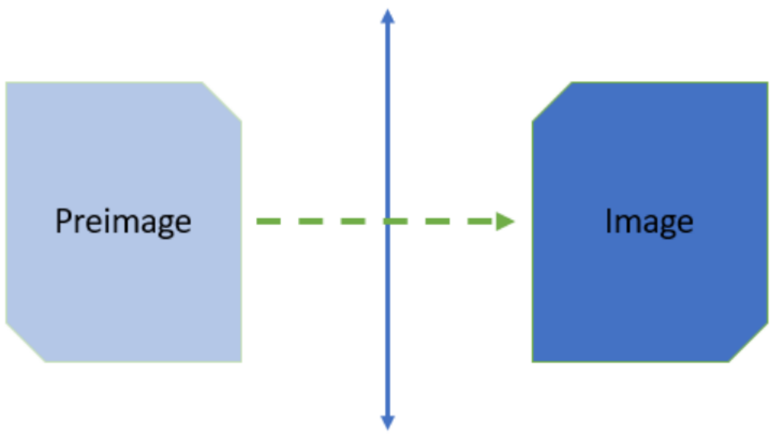


Figure 7: https://www.calculushowto.com/preimage-image/

- Point
  - A point is an exact location. It has no size, only position. ([Point (mathsisfun.com)](https://www.mathsisfun.com/geometry/point.html))

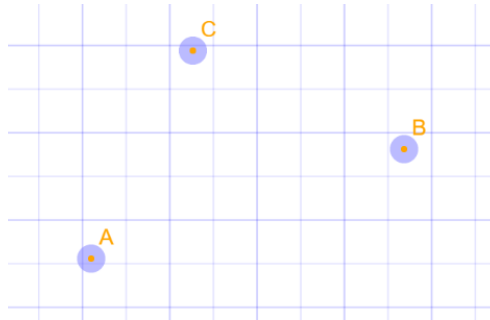


Figure 8: https://www.mathsisfun.com/geometry/point.html

- Vertex
  - A point where two or more line segments meet. A corner. ([Vertex Definition (Illustrated Mathematics Dictionary) (mathsisfun.com)](https://www.mathsisfun.com/definitions/vertex.html))

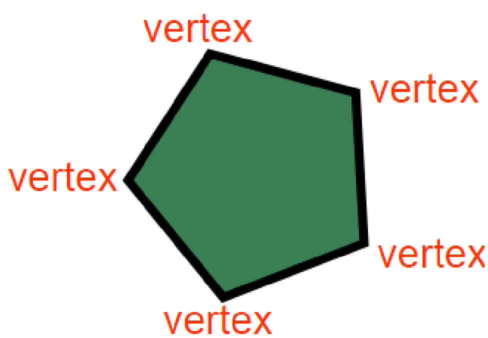


Figure 9: https://www.mathsisfun.com/definitions/vertex.html

- Angle
  - The amount of turn between two lines around their common point (the vertex). ([Angle Definition (Illustrated Mathematics Dictionary) (mathsisfun.com)](https://www.mathsisfun.com/definitions/angle.html))

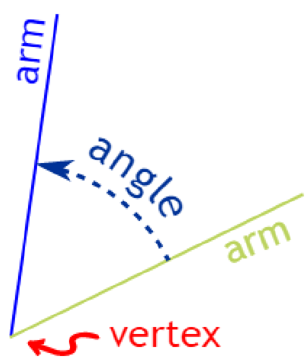


Figure 10: https://www.mathsisfun.com/definitions/angle.html

Materials

- Patty Paper
- TV or projector
- Whiteboard
- Document Camera
- Transformation Coordinate Rules (Optional)
- Transformation Coordinate Rules Key (Optional)
- Rotations Exit Ticket
- Rotations Exit Ticket Key
- All Turned Around Worksheet
- All Turned Around Worksheet Key
- Warmup Worksheet

Figure 11: Patty Paper
https://www.bing.com/images/search?view=detailV2&ccid=aCgTsL1G&id=CD6E17A22E2709707EAC606F4E75D7523B8887A5&thid=OIP.aCgTsL1G8TPUS3TrsQgx3gHaHZ&mediaurl=https%3a%2f%2fi5.walmartimages.com%2fasr%2f31adcaf7-5c3b-40b2-9059-29c4a21e4486.96b01615d10922bd4aaf

Instruction

Introduction (20 minutes)

Warmup (10 minutes)

- Hand out the Warmup Worksheet.
- Instruct students to draw the reflection of the word GEOMETRY over the line beneath it. The students must then describe the symmetry of each letter in this word.

GEOMETRY

- After students have had time to work, select a student to draw their representation of the reflection on the board and then call on non-volunteers to identify which letters have symmetry and describe the symmetry.
- Here are some questions to ask the students during the warmup:
    - What is the line of reflection?
    - Which letters are symmetric?
    - What is their line of symmetry?
    - How do you know?

Introducing Activity (10 minutes)

- Ask the students: “Of all the students in the class, who woke up the earliest?”
- Once the class determines which student woke up the earliest and what time they woke up, put that time up on a “wall” clock on the TV (or project it on the board).

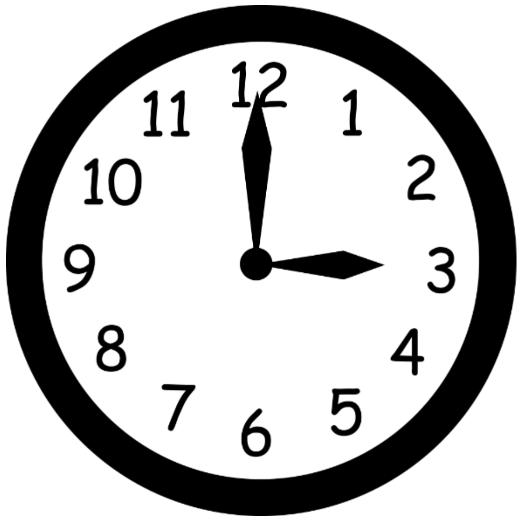


Figure 12: http://www.clipartbest.com/digital-clock-clip-art

- Then have the students determine who woke up next.
- Ask the question, “How many degrees do we move the minute hand to get from the first alarm clock time to the second?”
- Do a Think-Pair-Share, and ask one student to answer the question.
- Then, ask the students to describe the difference between clockwise and counterclockwise rotation.
- Once a volunteer has described these directions of rotation, ask the students to predict the location of the hand after a 90-degree counterclockwise rotation.
- Allow students to have individual think time as well as a minute to confer with their neighbors before a volunteer shares their thoughts.
- As a whole class, answer and discuss the question, “Where is the center of rotation for the minute hand?”
- Here are some questions to ask the students during the introduction to the activity:
    - What does a 90-degree angle look like?
    - How many minutes does that correspond to on the clock?
    - How many degrees do 5 minutes correspond to?
    - What is the center of rotation?
    - What is the difference between clockwise and counterclockwise?
    - Which one is the “positive” direction?

Activity (40 minutes)

Part One (10 minutes)

- Show the image below of a triangle on the coordinate plane and have students talk to their neighbors about how they think the teacher could use patty paper to rotate the triangle 90 degrees counterclockwise about the origin.

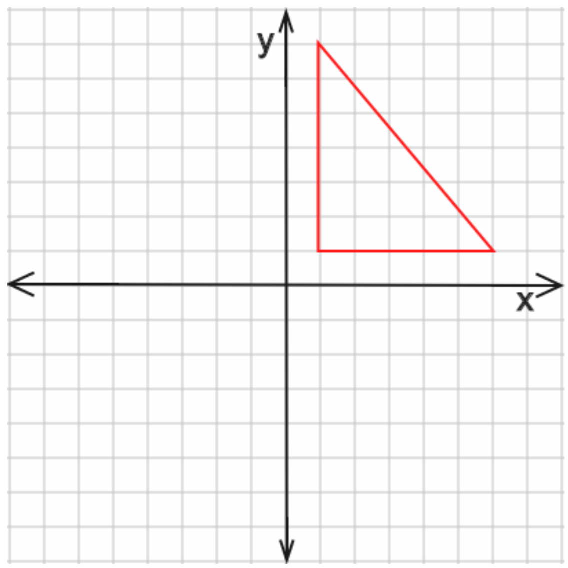


Figure 13: https://www.varsitytutors.com/ssat_middle_level_math-help/how-to-find-a-triangle-on-a-coordinate-plane

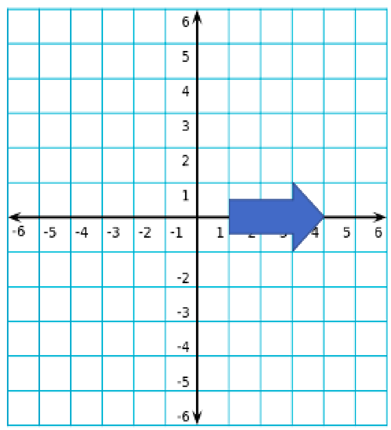
- Have both volunteers and non-volunteers describe their procedures, and have one student perform the procedure underneath the document camera.

- Next, have a whole class discussion of how they think they could write the algebraic notation for this rotation.
- Select a student to write their answer on the board.
  - The answer is: (x,y) → (y, -x).
- Do a Think-Pair-Share to discuss how to use this notation for a 90-degree counterclockwise rotation to find the functions for 180- and 270- degree counterclockwise rotations.
- Optional: Have the students fill out the Transformation Coordinate Rules.
  - This can help students in the future with how to do transformations on coordinate planes.
  - Note: There are other transformations on the page, so fill out the rotations section.

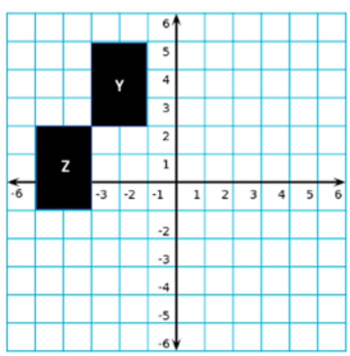
- Here are some questions to ask the students during this part of the activity:
  - How can we use what we learned yesterday about patty paper to help us perform rotations?
  - How can we use the function for a 90-degree turn to find one for a 180- and 270- degree turn?
  - How does 90 relate to 180?
  - How many 90 degree turns are in one 180 degree turn?

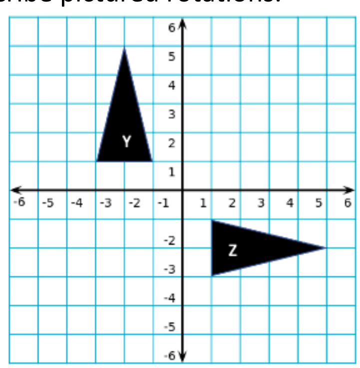
Part Two (20 minutes)

- Have students work in groups to complete the All Turned Around worksheet.
  - Question 1 requires students to predict the direction an arrow is pointing after certain rotations.

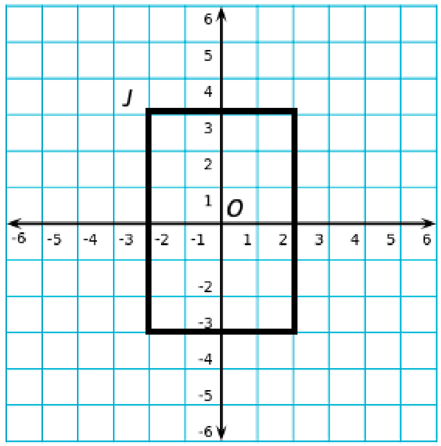


  - For question 2, they can use the patty paper to help them draw these rotations.
  - Question 3 asks the students to describe pictured rotations.

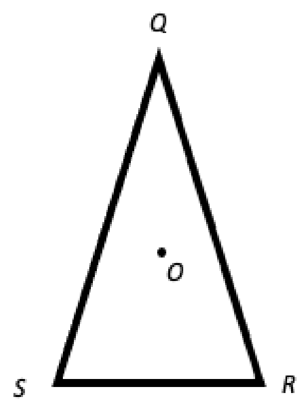




- Questions 4-5 ask students to draw a 45-degree rotation of a square and connect this rotation to the degree of the angle formed by the pre-image, center of rotation, and image of a point.



- Question 6 then explores rotations off the coordinate plane.

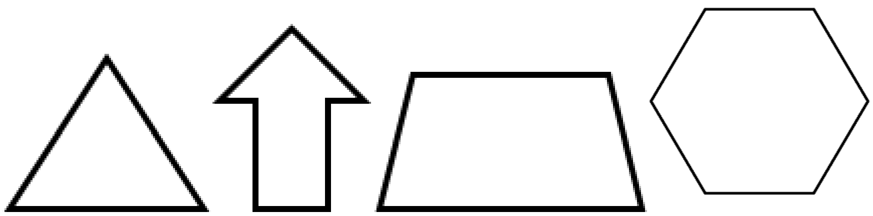


- During this group work time, monitor the students. Select students to present their work on certain problems during the whole class discussion.

- Here are some questions to ask the students as they work on the worksheet (each number refers to the question number on the worksheet):
  - 1. If this arrow was the hand of a clock, what would a rotation of 90 degrees look like? How many minutes correspond to a rotation of 90 degrees?
  - 2. How can you use the patty paper to draw the rotations? Can you do it without patty paper?
  - 3. Where would the center of rotation be? How would you use patty paper to perform these rotations?
  - 4. How is a rotation of 45 degrees different from a rotation of 90 degrees?
  - 6. How does this angle relate to the angle of rotation?

Part Three (10 minutes)

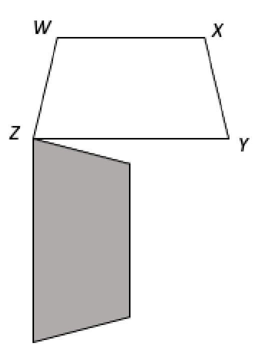
- After students present their solutions to 1-6, have the whole class walk through problems 7 and 8 together, which focus on rotational symmetry.
    - For number 7, the students must connect the concept of symmetry to rotations using a square.
    - Number 8 asks the students to identify both rotational and reflectional symmetry in figures.



- As a class, discuss what it means for a figure to be "mapped onto itself."
- Select and sequence the students' work for the whole-class discussion in order to structure the logic of the concepts and to highlight good student thinking.
- Here are some questions to ask the students as they work on the worksheet (each number refers to the question number on the worksheet):
    - 7. How is this similar to what we did with reflections?
    - 8. How do the angles of rotational symmetry and the number of lines of reflectional symmetry relate to one another? How does the angle of rotational symmetry relate to the number of sides/angles in the polygon?

Conclusion (10 minutes)

- Have students complete an exit ticket.
    - The first problem asks the students to describe the rotation of a given picture, label the new vertices of the image, and find a specified angle.
    - The next problem asks the students to describe a shape that has rotational symmetry of 30 degrees.



Differentiation

Grouping:

- Students will be grouped by the arrangement of their desks into groups of 3-4 students. These groups will be mixed ability grouping for peer support and collaborative learning.
- The peer interaction will act as a means of differentiation. The heterogeneous groups will allow for more advanced students to solidify their understanding as they support their peers, and students who need extra support will have guidance from their group members.

Students who struggles with algebra:

- A guide for using function notation will be on the board. It will say “(horizontal input, vertical input) → (horizontal output, vertical output).”
- If the students are struggling with algebraic notation, they will be encouraged to make a table of values for the input and output and identify any patterns between the two.
- Student can lean on the visuals presented in the class and will be encouraged to try to draw figures if one is not given.
- They will be supported by the use of patty paper to physically represent rotations.

Students who processes information more slowly than their peers:

- Students will benefit from peer support during the Think-Pair-Share and the group work.
- Students’ group will be directed to read instructions to the problems out loud and discuss what the question is asking before they begin working.
- The chunking of the problems will also support students by not overwhelming them with information.

Students who need an extra challenge:

- During the group work, these students may work at an accelerated pace and do the second set of problems (7 and 8) if they finish before their peers.
- If they finish the problems, they will be directed to the challenge problem.

Assessment

Formative Assessment

- The warm up will show the teacher if students can reflect objects over a horizontal line and if they can identify the line of reflection. It will also tell the teacher if the students understand that if objects are symmetrical, then they will look the same after being reflected across their line of symmetry.
- The class discussions and group work will show the teacher if students understand how to use patty paper to perform reflections. It will also show if they know what a 90-degree and 45-degree angle is as well as if they can differentiate between clockwise and counterclockwise. Whatever concepts and procedures the students have difficulty with during the group work will be discussed during the whole group discussion so that any misconceptions are corrected and challenging ideas are reinforced.

Summative Assessment

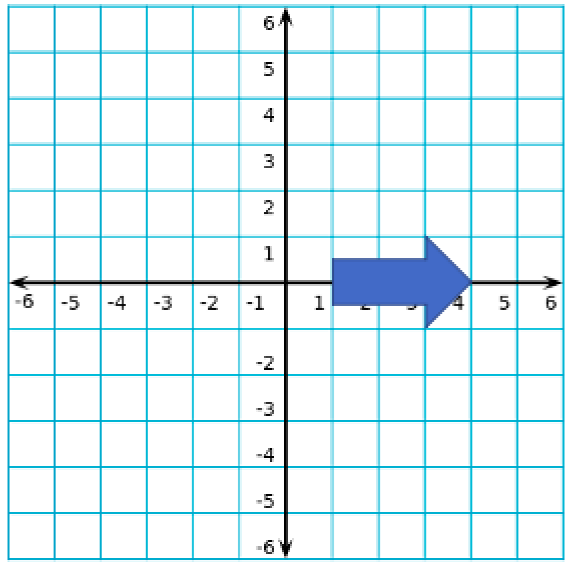
- The exit ticket will show the teacher if students can interpret a visual representation of a translation and describe it in words. It will also show if they understand that the angle of rotation is the same as the angle between a point, the center of rotation, and the points' image. Lastly, the exit ticket will show if students understand the requirements for rotational symmetry.

Warmup Worksheet Name:___________________________

Draw the reflection of the word below. Then describe the symmetry of each letter in the word.

GEOMETRY

All Turned Around



1) Predict the direction of the arrow after the following rotations. The center of rotation for each is (0,0).

a. 90° counterclockwise

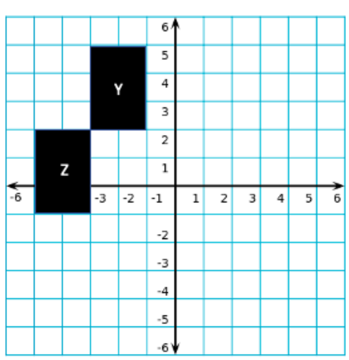
b. 90° degrees clockwise

c. 180° degrees

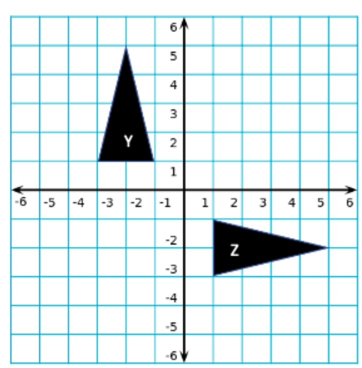
2) Draw the rotations from each part of Question 1. Were you correct?

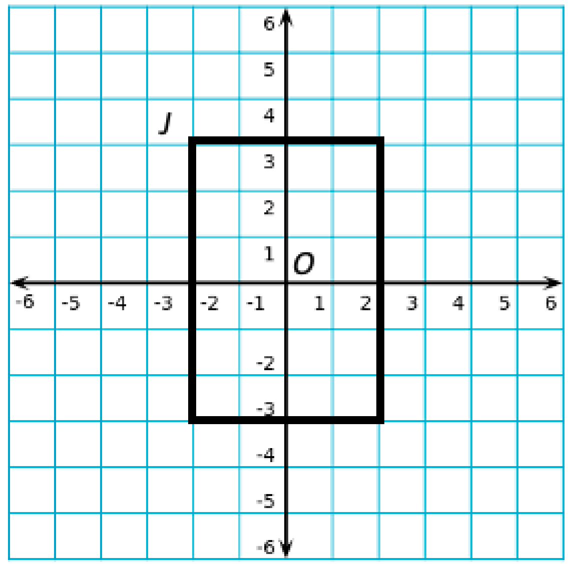
3) Describe a rotation that would map figure Y onto figure Z. (Hint: Identify the center of rotation first.)

a.



b.



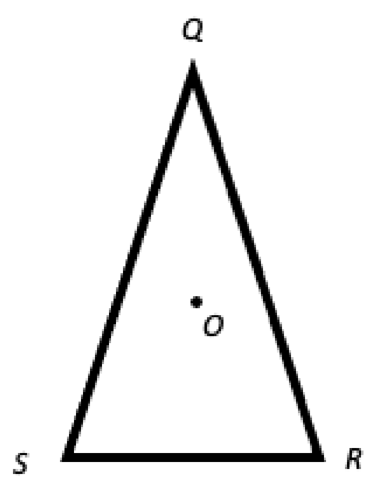


4) Draw a rotation of the rectangle 45° counterclockwise about the origin. Label the point *J′* that is the image of point *J*.

5) What is the measure of *∠JOJ′*?

6) The diagram below shows isosceles triangle *QRS* and point *O* inside of it. Draw the following rotations. Label the images of the 3 vertices.

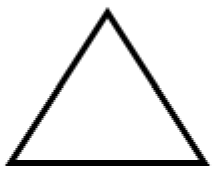
a) Rotate 90° counterclockwise around the origin
b) Rotate 90° clockwise around point *S*
c) Rotate 90° counterclockwise around point *Q*



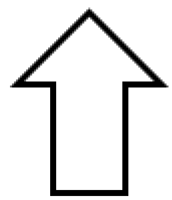
7) How do rotations relate to symmetry? (Hint: Can rotations map a square onto itself?)

8) Determine whether these figures have reflectional symmetry, rotational symmetry, or both. Identify the lines of reflectional symmetry and the angles of rotational symmetry.

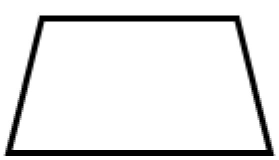
a) Equilateral Triangle



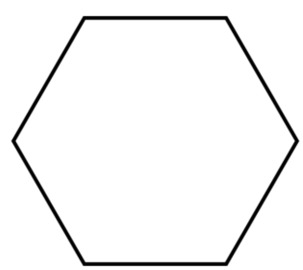
b) Arrow



c) Trapezoid



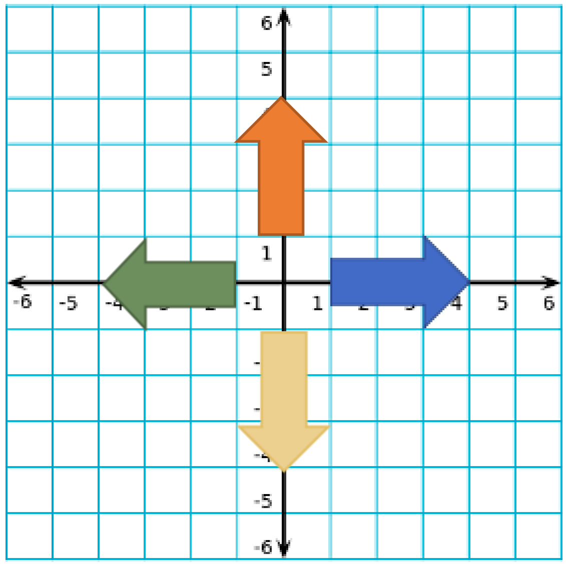
d) Regular Hexagon



Challenge Problem:

Do you think it is possible for a figure to have an angle of rotational symmetry of 37°? If so, describe the figure. If not, explain why not.

All Turned Around KEY



1) Predict the direction of the arrow after the following rotations. The center of rotation for each is (0,0).

a. 90° counterclockwise Answers will vary.

b. 90° degrees clockwise Answers will vary.

c. 180° degrees Answers will vary.

2) Draw the rotations from each part of Question 1. Were you correct?

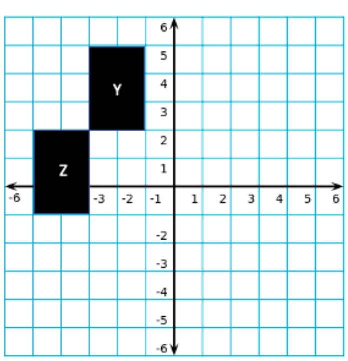
a. orange arrow

b. yellow arrow

c. green arrow

3) Describe a rotation that would map figure Y onto figure Z. (Hint: Identify the center of rotation first.)

a.

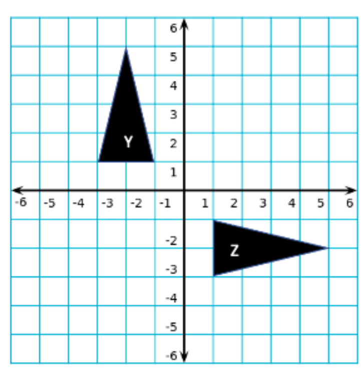


Center of rotation:

(-3, 2)

Rotated 180 degrees clockwise/ counter clockwise

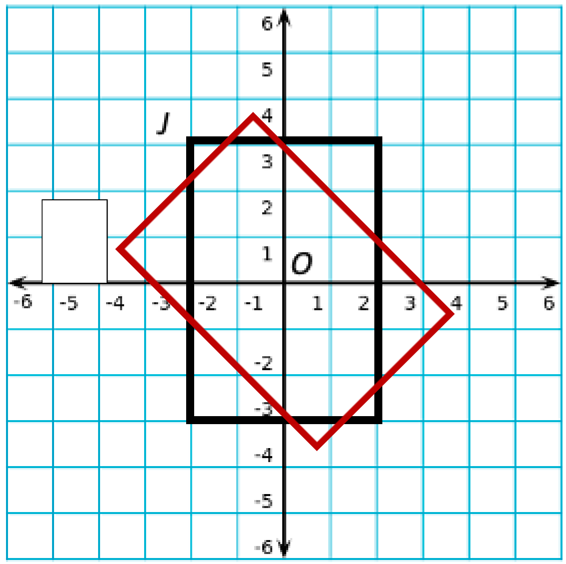
b.



Center of rotation:

(-2, -2)

Rotated 120 degrees clockwise/ 240 degrees counter clockwise



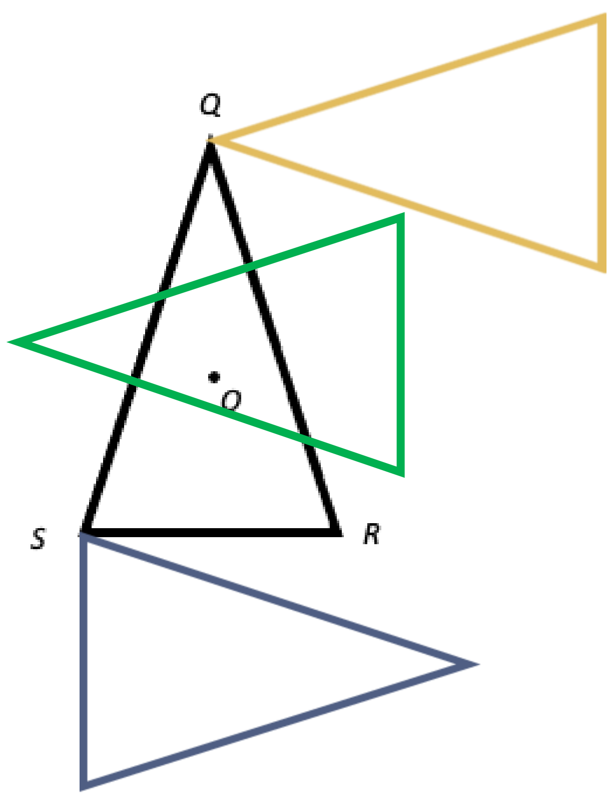
4) Draw a rotation of the rectangle 45° counterclockwise about the origin. Label the point *J′* that is the image of point *J*.

5) What is the measure of ∠*JOJ′*?

45 degrees

6) The diagram below shows isosceles triangle *QRS* and point *O* inside of it. Draw the following rotations. Label the images of the 3 vertices.

a) Rotate 90° counterclockwise around the origin green triangle
b) Rotate 90° clockwise around point *S* blue triangle
c) Rotate 90° counterclockwise around point *Q* yellow triangle

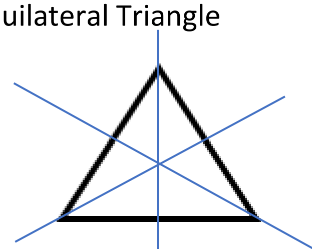


7) How do rotations relate to symmetry? (Hint: Can rotations map a square onto itself?)

Answers may vary.

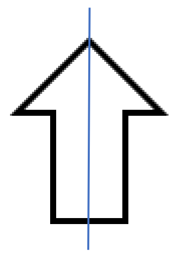
8) Determine whether these figures have reflectional symmetry, rotational symmetry, or both. Identify the lines of reflectional symmetry and the angles of rotational symmetry.

a) Equilateral Triangle



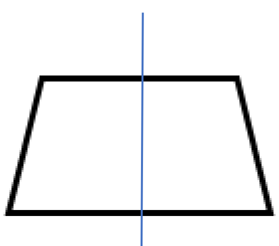
both, 120 degrees

b) Arrow



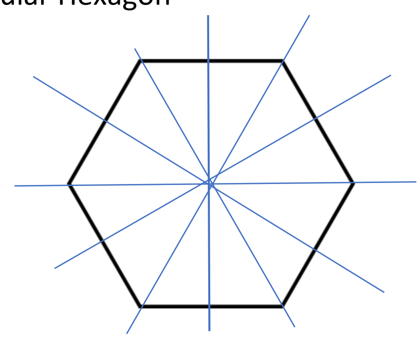
reflectional symmetry

c) Trapezoid



reflectional symmetry

d) Regular Hexagon



both, 45 degrees

Challenge Problem:

Do you think it is possible for a figure to have an angle of rotational symmetry of 37°? If so, describe the figure. If not, explain why not.

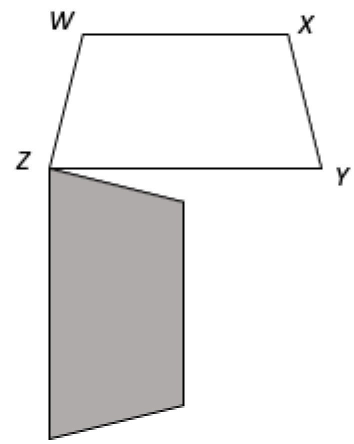
Answers may vary.

Example answer: No because 360 degrees/ 37 degrees is not a whole number, so drawing a figure with 9.72 sides would not be possible.

Rotations Exit Ticket

Name: ______________________

1. The figure shows trapezoid *WXYZ* and its image (shaded) after a rotation.
   a. Describe the rotation in words.



   b. Label the vertices *W'*, *X'*, and *Y'* of the rotated figure.

   c. If $m\angle WZY = 75°$, find $m\angle W'ZY$.

2. Describe a figure that has an angle of rotational symmetry of 30°.

Rotations Exit Ticket

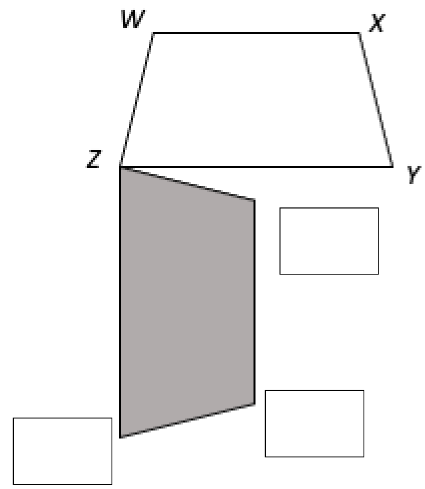
Name: __KEY_______________

1. The figure shows trapezoid *WXYZ* and its image (shaded) after a rotation.
   a. Describe the rotation in words.

      Answers will vary.

      Example answer:

      The center of rotation is Z and 90 degrees clockwise.



   b. Label the vertices *W'*, *X'*, and *Y'* of the rotated figure.

   c. If m∠*WZY* = 75°, find m∠*W'ZY*.

90 degrees – 75 degrees = 15 degrees

Answer: 15 degrees

2. Describe a figure that has an angle of rotational symmetry of 30°.

Answers may vary.

Example answer:

360 degrees / 30 degrees = 12

Answer: Regular Dodecagon

Transformation Coordinate Rules

Translations

Reflections

Over the x-axis:

Over the y-axis:

Over the line y = x:

Over the line y = -x:

Rotations

Center of rotation (0,0)

90° counterclockwise or ____ clockwise turn:

180° counterclockwise or ____ clockwise turn:

270° counterclockwise or ____ clockwise turn:

Transformation Coordinate Rules KEY

Translations

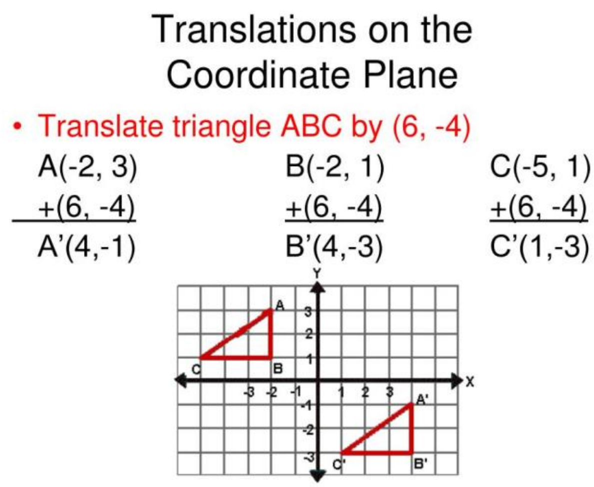


Figure 1: https://www.slideserve.com/theola/translations-on-the-coordinate-plane

Reflections

Over the x-axis: (x, y) → (x, -y)

Over the y-axis: (x, y) → (-x, y)

Over the line y = x: (x, y) → (y, x)

Over the line y = -x: (x, y) → (-y, -x)

Rotations

Center of rotation (0,0)

90° counterclockwise or 270 degrees clockwise turn:

$(x,y) \rightarrow (-y, x)$

180° counterclockwise or 180 degrees clockwise turn:

$(x,y) \rightarrow (-x, -y)$

270° counterclockwise or 90 degrees clockwise turn:

$(x,y) \rightarrow (y, -x)$