Graphing the Trigonometric Functions

Submitted by: Maria Rhodes, Math
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Target Grade: 11th grade Trigonometry

Time Required: 90 minutes

Standards

Common Core Math Standards

- CCSS.MATH.CONTENT.HSF.TF.B.5
  Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Lesson Objectives

Students will:

- Graph sine, cosine, cosecant, and secant using the unit circle.
- Identify the period, amplitude, and asymptotes of a graph.

Central Focus

In this lesson students will explore how to graph trigonometric graphs and identify the period, amplitude, and asymptotes. Students will learn to recognize the graphs which will be important in future lessons as well as in other disciplines. The lesson will launch by recognizing sinusoidal waves in the real world such as visible waves. Then students will examine parts of a sine wave such as the period, amplitude, and asymptote which will allow them to identify the parts of cosine, cosecant, secant, tangent, and cotangent graphs.

Key Terms: period, asymptote, amplitude, trigonometry, sine, cosine, cosecant, secant, tangent, cotangent

Background Information

Coming into the lesson, the students should be familiar with term trigonometry and the concepts that fall in its domain. Specifically for this lesson, students should understand what each of the trigonometric functions are (sine, cosine, etc.). They need to know the units of angle measures, which are degrees and radians, and be familiar with working with the variable theta. Because the lesson consists of graphing based on the unit circle, the students should understand radians as a unit angle measure. Additionally, they should know what domain and range are with respect to both the unit circle and graphs.

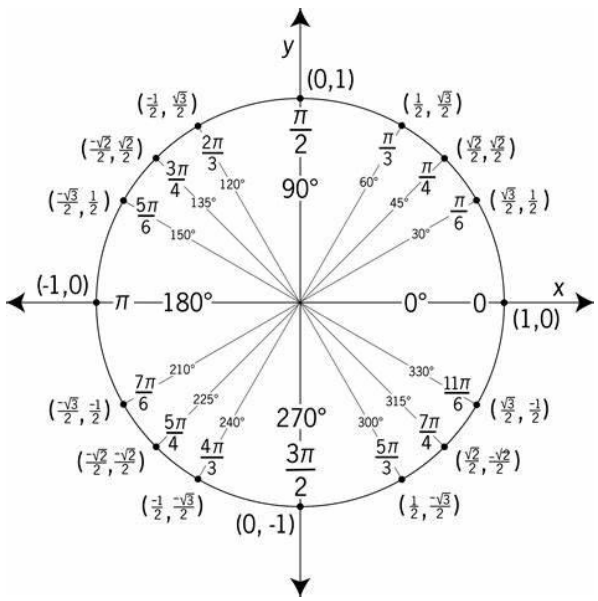


Figure 1: http://etc.usf.edu/clipart/43200/43215/unit-circle7_43215.htm

Prior to this lesson introduces new terms addressing the parts of trigonometric graphs. They will learn the new terms period, asymptote, and amplitude.

- Period
    - The length from one peak to the next (or from any point to the next matching point) of a periodic function.
    - In other words the length of one full cycle. (Period Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

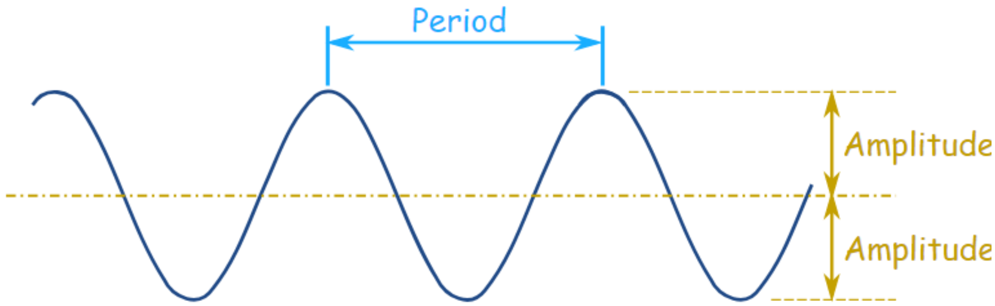


Figure 2: https://www.mathsisfun.com/definitions/period.html

- Asymptote
    - A line that a curve approaches as it heads towards infinity. (Asymptote Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

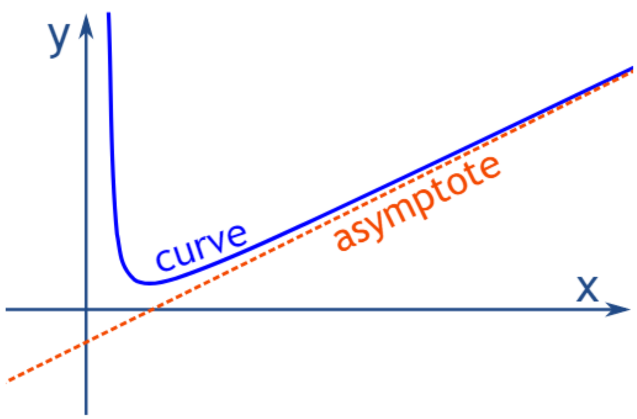


Figure 3: https://www.mathsisfun.com/definitions/asymptote.html

- Amplitude
  - The height from the center line to the peak (or trough) of a periodic function.
  - Or we can measure the height from highest to lowest points and divide that by 2. ([Amplitude Definition (Illustrated Mathematics Dictionary) (mathsisfun.com)](https://www.mathsisfun.com))

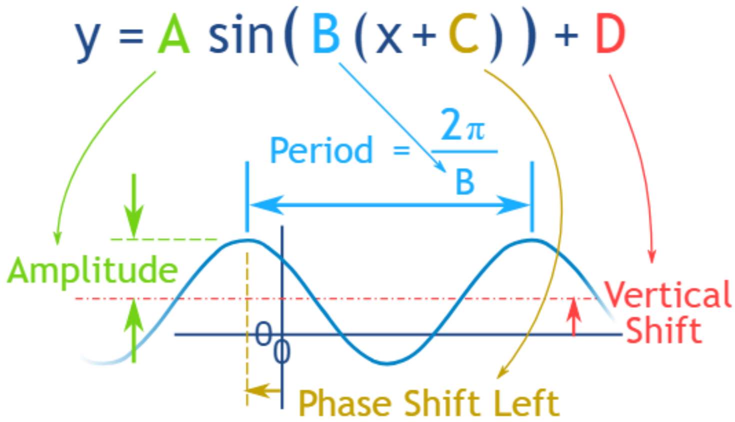


Figure 4: https://www.mathsisfun.com/algebra/amplitude-period-frequency-phase-shift.html

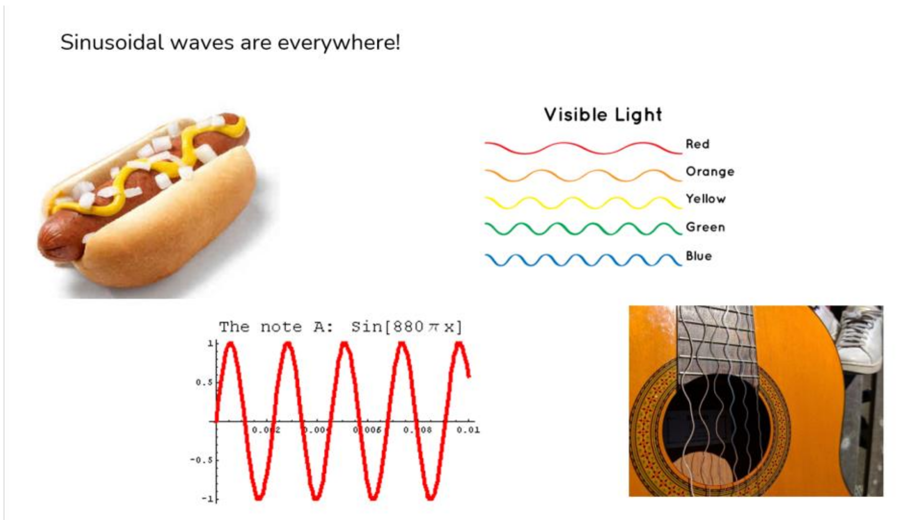
Materials

- Computer
- Projector
- Graphing the Trigonometric Function Worksheet
- Graphing the Trigonometric Function Worksheet Key for each group
- Graphing the Trigonometric Function Exit Ticket
- Graphing the Trigonometric Function Exit Ticket Key
- Slinky
- Rulers for graphing

Instruction

Introduction

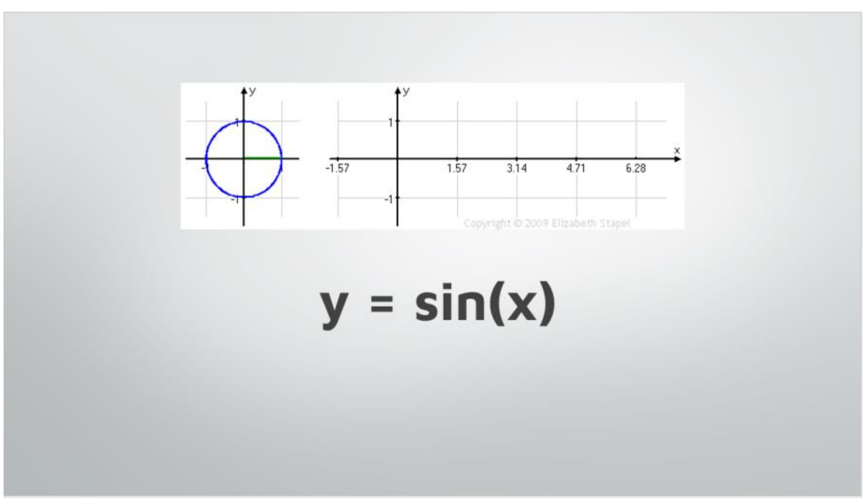
- Begin with a discussion of where the students have seen sinusoidal functions in real life.
- Have the students share examples.
- After the students share examples, look at pictures on the slide together.



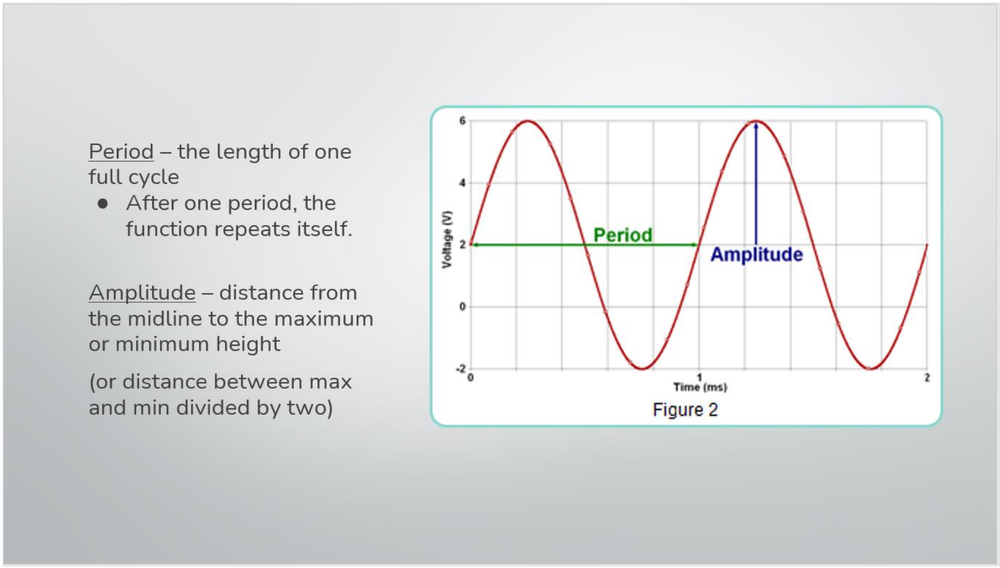
Activity

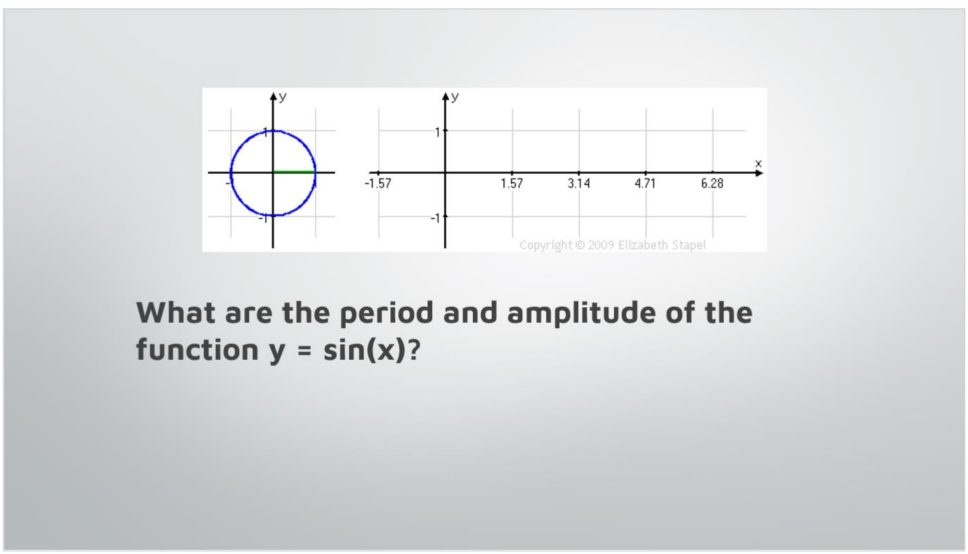
Part One

- Using the powerpoint, review the graph of sine.
- Handout a sheet of paper and have the students draw the graph of sine.



- Ask students to volunteer explanations for how they went about graphing the sine function.
- Discuss the new terms period and amplitude and identify them in the graph of sine.





- To give a real-life visual of the period and asymptotes, do an activity in the hallway in which a slinky is used to produce a sinusoidal pattern.

- Have two students make waves with the slinky, changing the amplitude and period (how far they move the slinky and how fast).
    - If the halls are tiled, the tiles can be used to measure the period and amplitude.

Let's go make some waves!

- After this activity, do a think-pair-share activity in which they identify the parts of the graphs displayed on the board.

Part Two

- Create at least five groups.
- Have the students split up and use the table and graph provided to try and generate the graphs of the other 5 trigonometric functions.
- Assign each group a function to graph (noted in the picture below as well as the slide attached at the bottom).

What about the other trig graphs?

Group 1: y = cos(x)

Group 2: y = tan(x)

Group 3: y = sec(x)

Group 4: y = csc(x)

Group 5: y = cot(x)

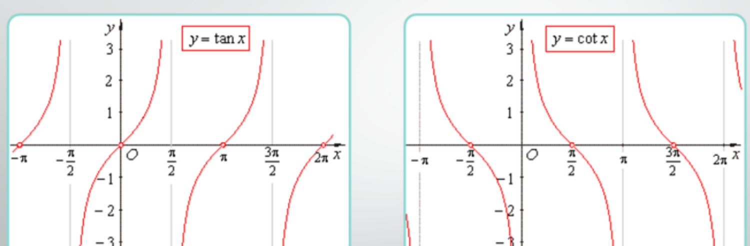
I will come around and check your graphs when you finish. When I give you the green light, you will draw your graph on a piece of chart paper and post it around the room.

- Give them a piece of chart paper on which the students will draw their final graph to hang up on the wall.
    - Accommodate for students when choosing which functions to give which groups.
    - During the group work, help the groups that have the graphs of tangent, cotangent, secant, and cosecant to help them understand how to graph the functions near their asymptotes.
    - If students finish early, give them a challenge, asking them a question such as, “What is the amplitude of y = 3cos(x)?”
    - Students can also work on the windmill problem if they have extra time.

Extension: Chasing the Wind

Windmills have been used for centuries as a tool to harness the power of the wind. In your groups, do some research to find out how big the blades of the average wind turbine are and how tall they stand from the ground. Using this information, graph the path of the windmill blades, assuming that it takes the blades 5 seconds on average to make a full rotation. Identify the period and amplitude of this graph.

- As the students work, walk around the classroom to check on the students as well as to help them.
- After they are given time to work together, have the class reconvene.
- The class will then be asked about the parts of the graph, including domain, range, period, amplitude, and asymptotes.



Where are the asymptotes?

Closure

- Hand out the exit ticket.
- Have the students work on the exit ticket as an evaluation of how well the students learned the material.
    - It includes identifying graphs and parts of graphs, as well as conceptually thinking about the terms.

Differentiation

Students with Learning Disabilities

- While they work, pay close attention to their work to check for understanding.
- They will not be expected to participate heavily in classroom discussions.
- The students will be given a choice of whether to work with another student during the think-pair-share and while solving the word problem.
- To set a clear outline and make the student comfortable, the teacher will foreshadow what activity and instructions will come next.

ELL Students

- Before class, provide the students with a short review of the required terms to know for the class.
- Increased use of visuals, which rely less on words, will also help them with their understanding of the topic.
- Put the ELL students in groups with students who have some level of skill in their native language.
- Provide a word bank available with definitions for these students.

Advanced Students

- Allow the students to move at a faster pace on the worksheet or provide the option to work alone.
- The students can help the people in their group that may be struggling.
- If the students finish early have them work on the extension problem.

Assessment

Formative Assessment

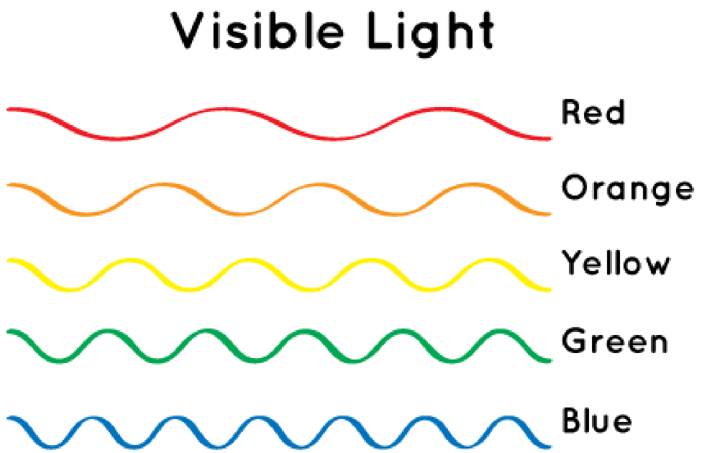
- The discussions will be key as formative assessments. Hearing the students' questions and seeing their work during the activities will allow the teacher to evaluate how the students are progressing.

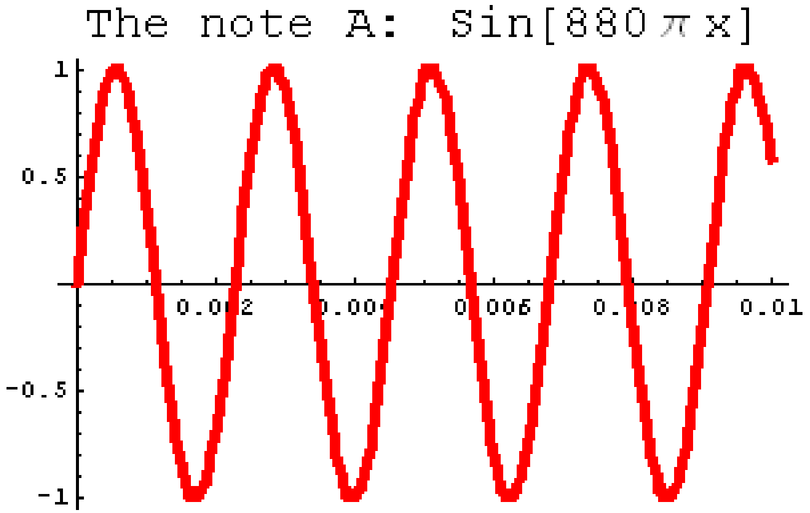
Summative Assessment

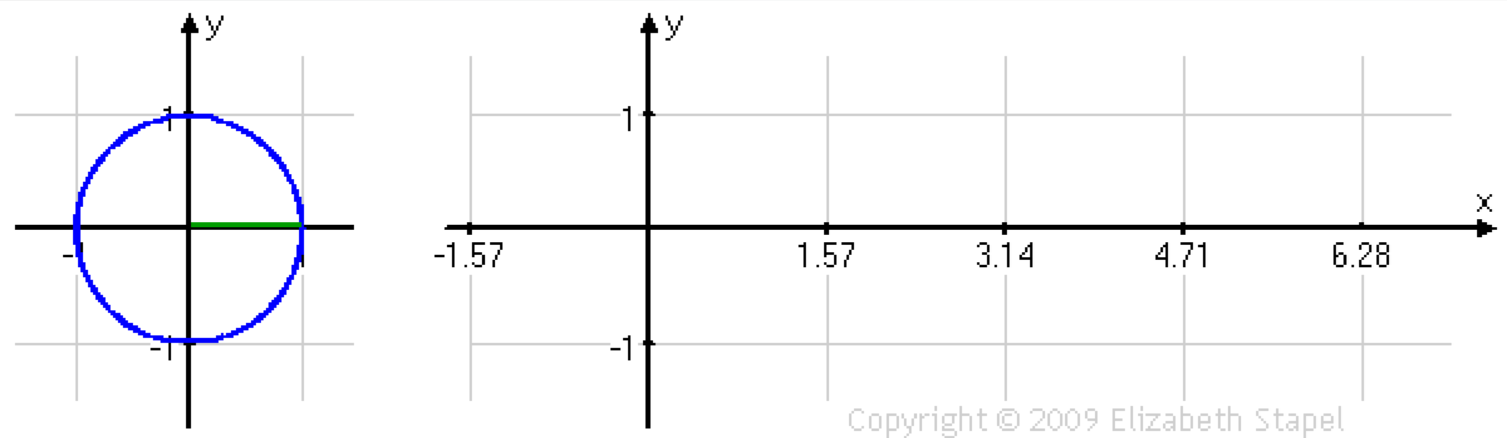
- The last activity/worksheet and the exit ticket will act as summative assessments. The worksheet and the exit ticket will allow the teacher to see what concepts the students understood as well as what concepts they may be struggling with.

Trigonometric Graphs

Sinusoidal waves are everywhere!






y
1
-1
y
1
-1
x
-1.57
1.57
3.14
4.71
6.28
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$y = \sin(x)$

Period – the length of one full cycle

- After one period, the function repeats itself.

Amplitude – distance from the midline to the maximum or minimum height

(or distance between max and min divided by two)

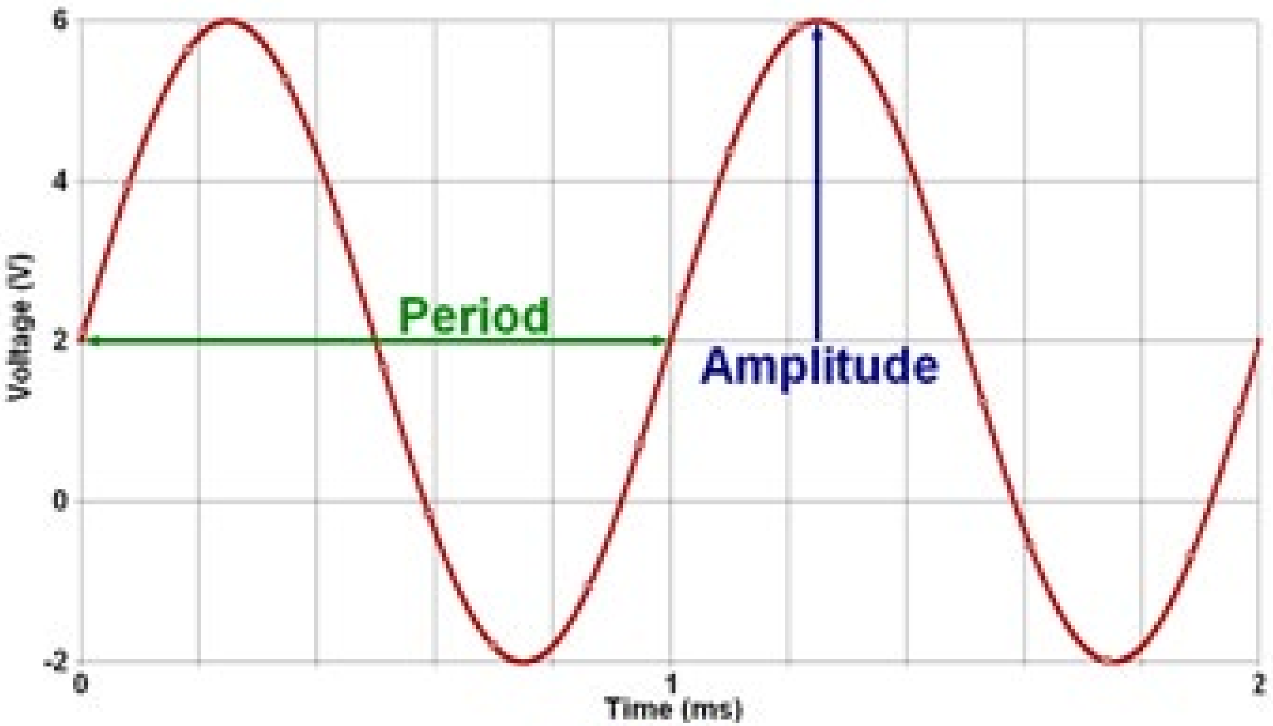
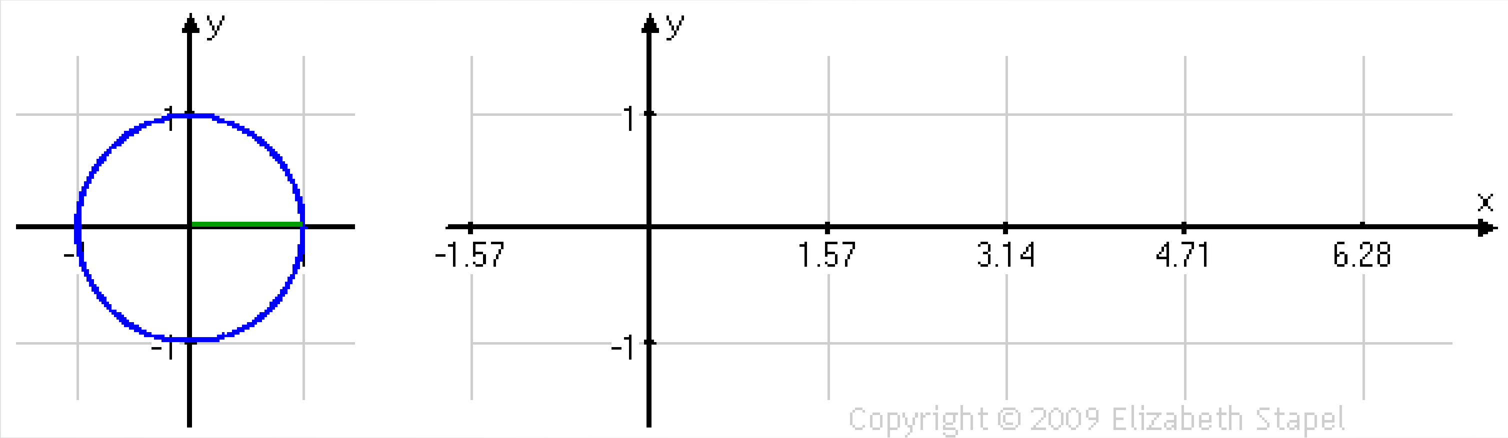


Figure 2



What are the period and amplitude of the function y = sin(x)?

Let’s go make some waves!

What about the other trig graphs?

Group 1: $y = \cos(x)$

Group 2: $y = \tan(x)$

Group 3: $y = \sec(x)$

Group 4: $y = \csc(x)$

Group 5: $y = \cot(x)$

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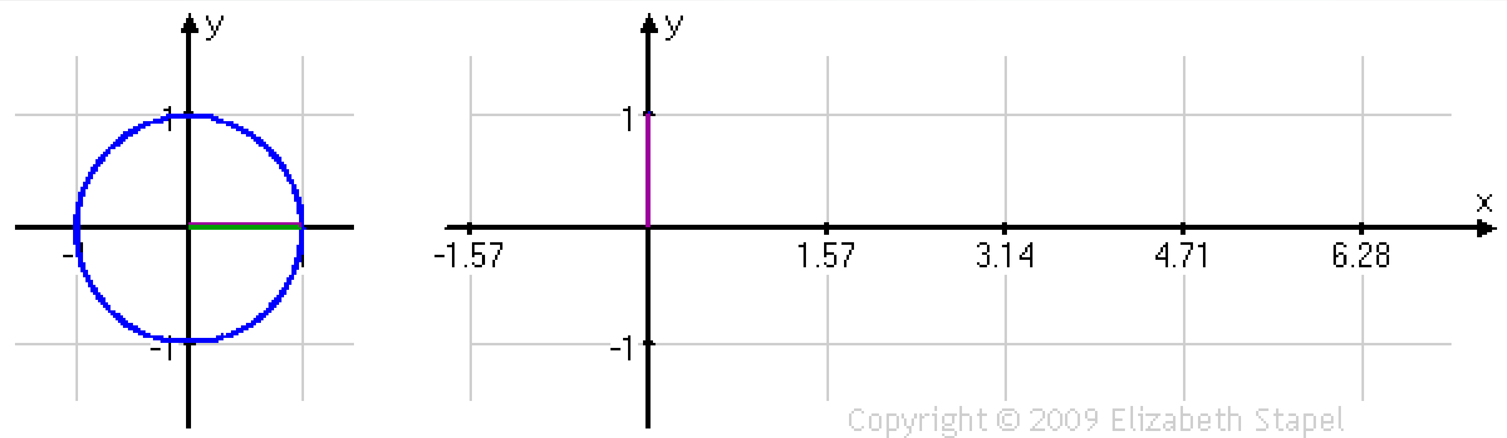
y
1
-1
y
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-1
x
-1.57
1.57
3.14
4.71
6.28
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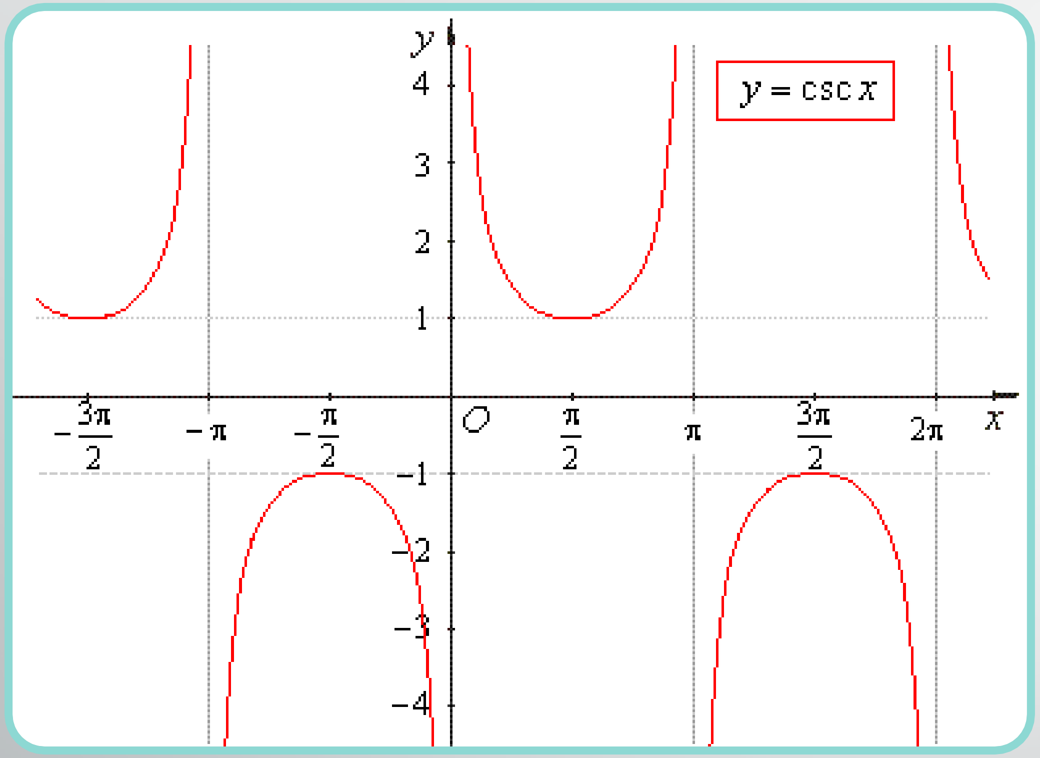
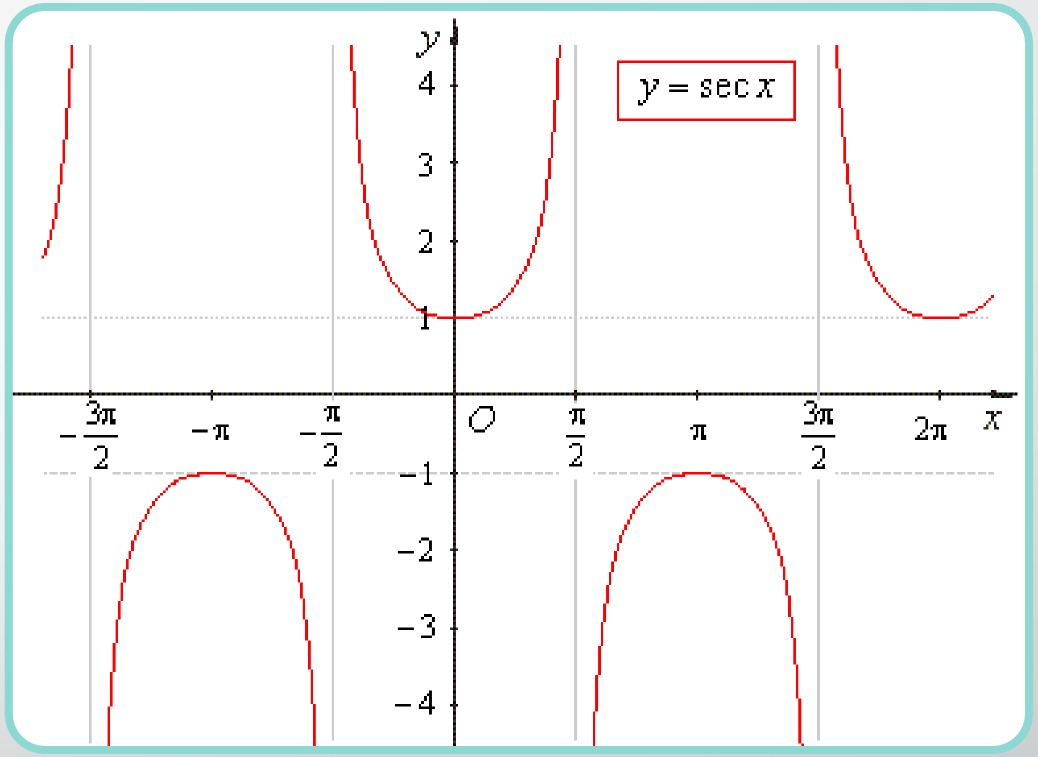
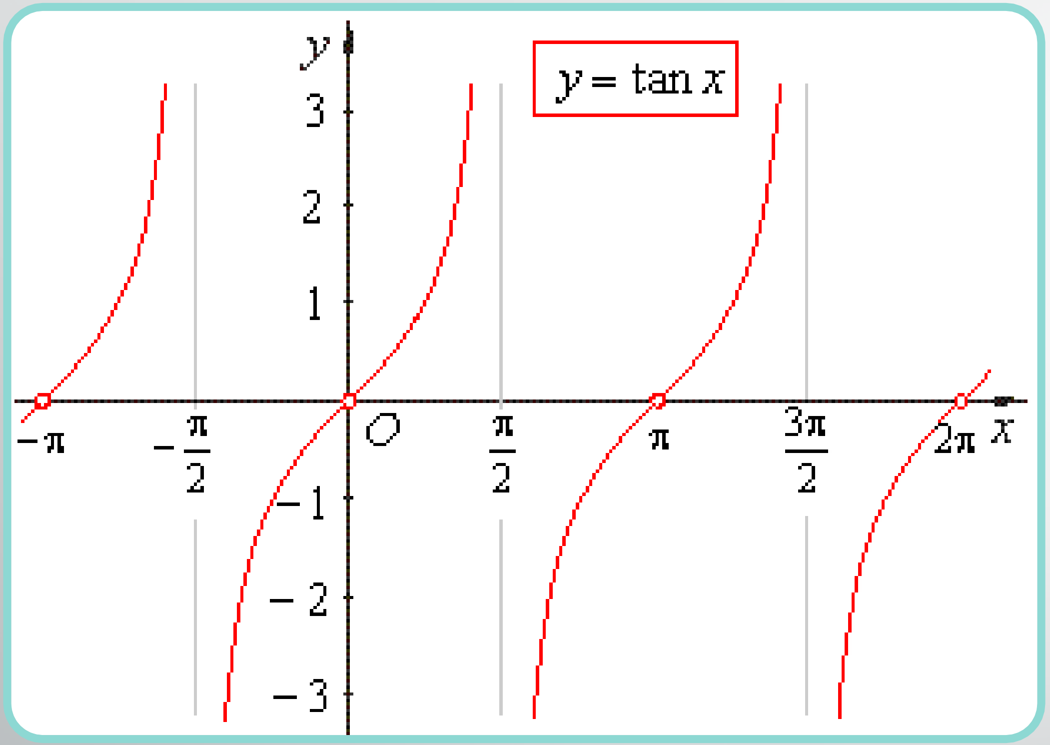
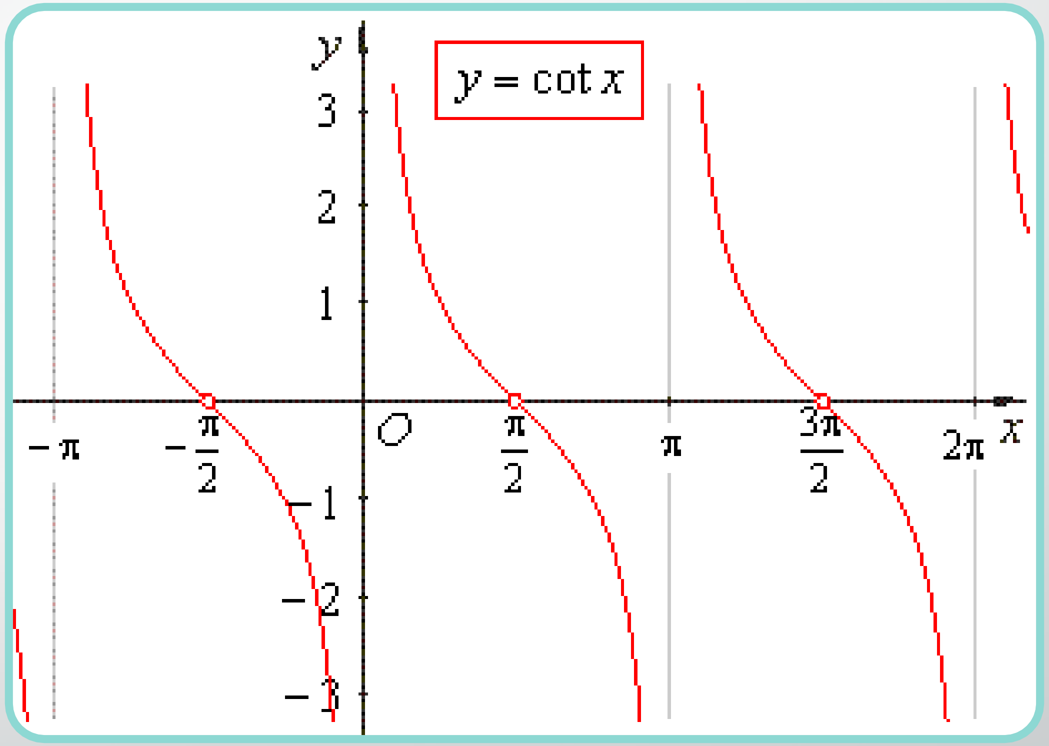
y = cos(x)


y = csc x
y
4
3
2
1
−1
−2
−3
−4
x
O
−3π/2
−π
−π/2
π/2
π
3π/2
2π


y = sec x
y
4
3
2
1
−1
−2
−3
−4
x
O
−3π/2
−π
−π/2
π/2
π
3π/2
2π

Asymptotes





Where are the asymptotes?

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Graphing the Trigonometric Function Worksheet   Name:

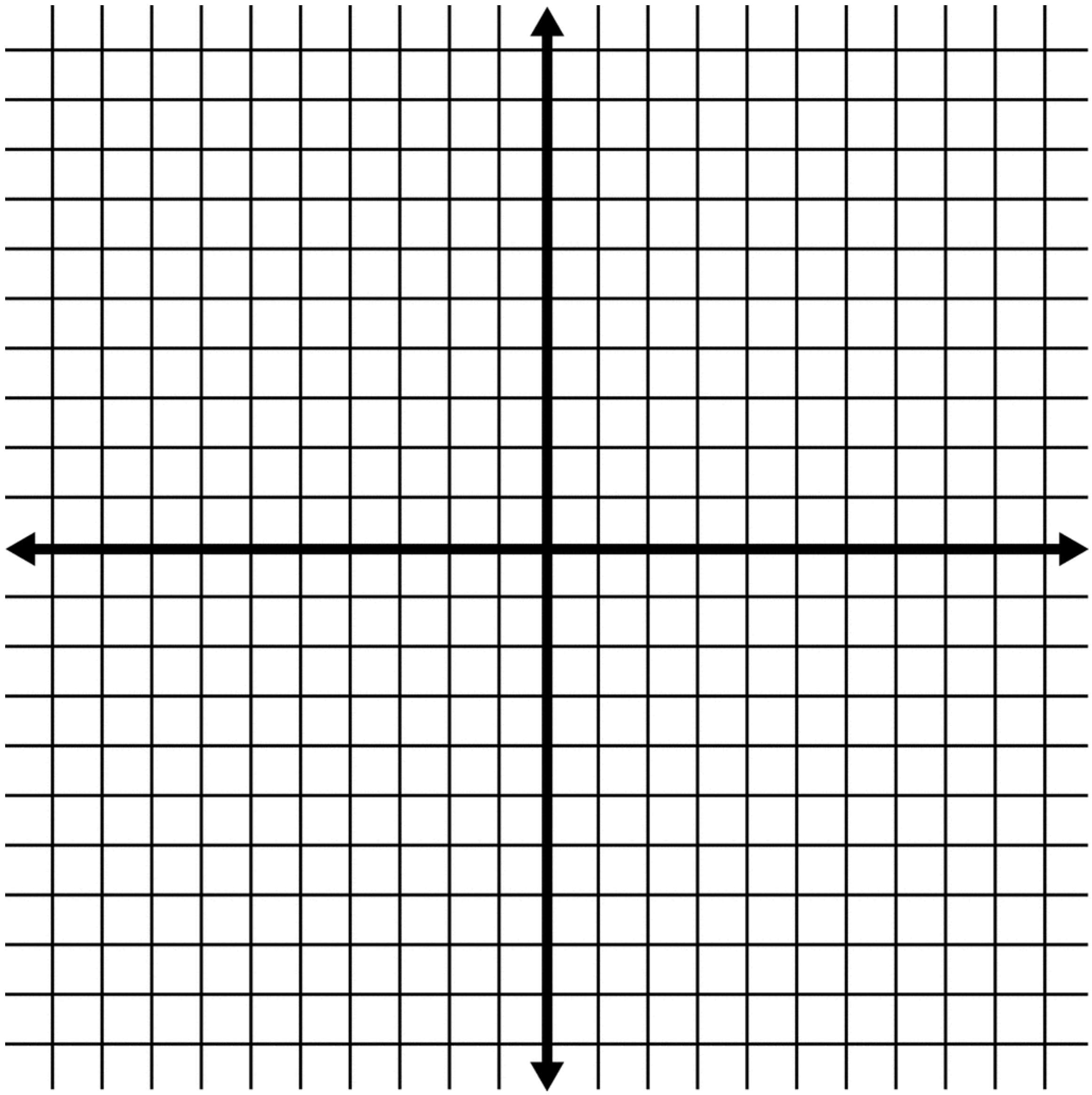
Write your given function:

1. What is the domain and range of your function?

2. If there are values not in your domain, what happens when you plug in numbers close to that value?

3. Using the unit circle, fill in the table below.
4. Use the table and the information from number 3 to graph your function.

x	y
0	
$\pi/6$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$5\pi/6$	
π	
$5\pi/4$	
$3\pi/2$	
$7\pi/4$	
2π	



Graphing the Trigonometric Function Worksheet Name: Group 1

Write your given function: y = cos(x)

1. What is the domain and range of your function?

   Domain: All real numbers

   Range: [-1, 1]

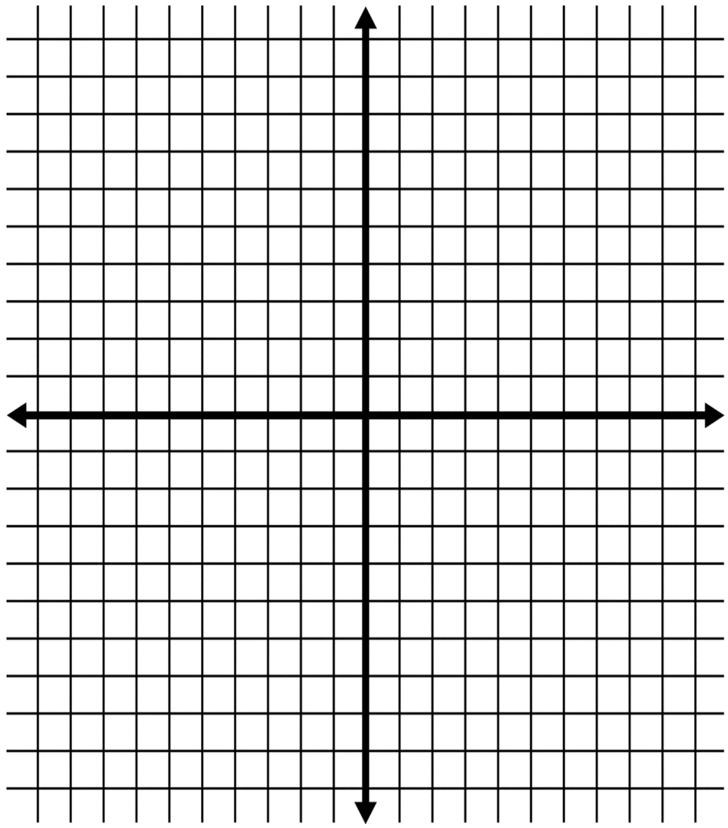
2. If there are values not in your domain, what happens when you plug in numbers close to that value?

   Answers may vary.

3. Using the unit circle, fill in the table below.

4. Use the table and the information from number 3 to graph your function.

x	y
0	1
π/6	$\frac{\sqrt{3}}{2}$
π/3	$\frac{1}{2}$
π/2	0
2π/3	$-\frac{1}{2}$
5π/6	$-\frac{\sqrt{3}}{2}$
π	-1
5π/4	$-\frac{\sqrt{2}}{2}$
3π/2	0
7π/4	$\frac{\sqrt{2}}{2}$
2π	1



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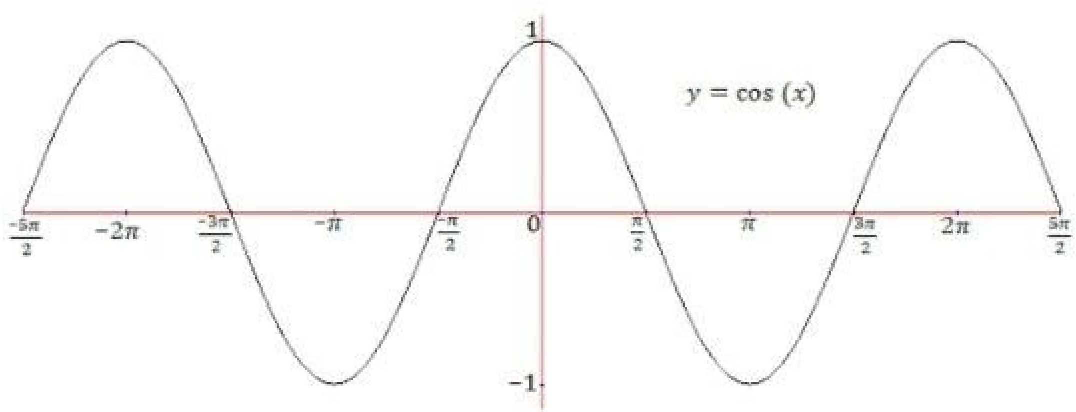


Figure 1: https://www.quora.com/What-is-the-range-of-cos-1-x

Graphing the Trigonometric Function Worksheet Name: Group 2

Write your given function: y = tan(x)

1. What is the domain and range of your function?

   Domain: All real numbers, i.e., $(-\infty, \infty)$

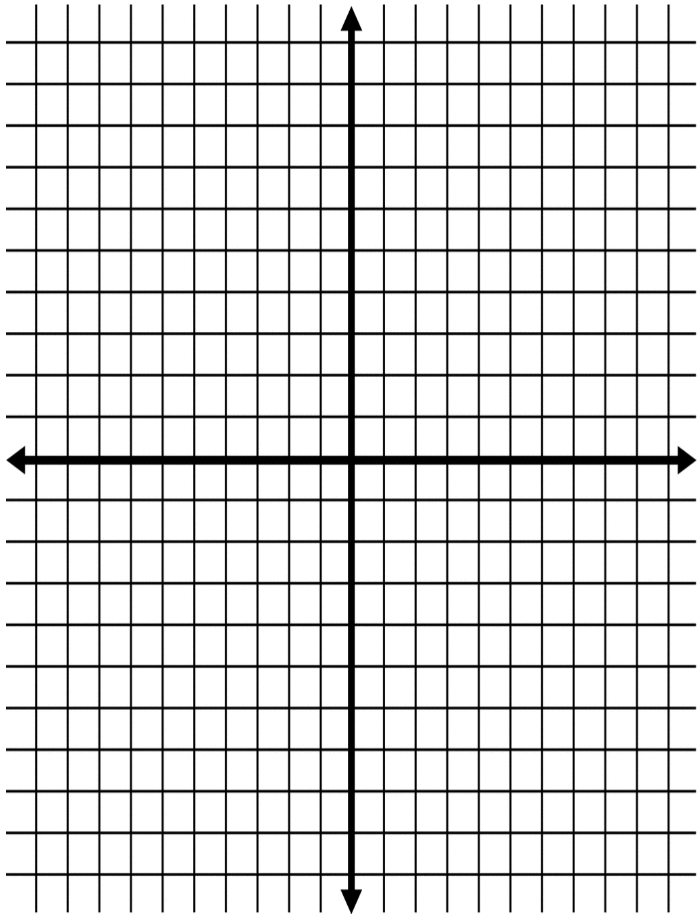
   Range: [-1, 1]

2. If there are values not in your domain, what happens when you plug in numbers close to that value?

   Answers may vary.

3. Using the unit circle, fill in the table below.
4. Use the table and the information from number 3 to graph your function.

x	y
0	0
π/6	$\frac{\sqrt{3}}{3}$
π/3	$\sqrt{3}$
π/2	undefined
2π/3	$-\sqrt{3}$
5π/6	$-\frac{\sqrt{3}}{3}$
π	0
5π/4	1
3π/2	undefined
7π/4	-1
2π	0



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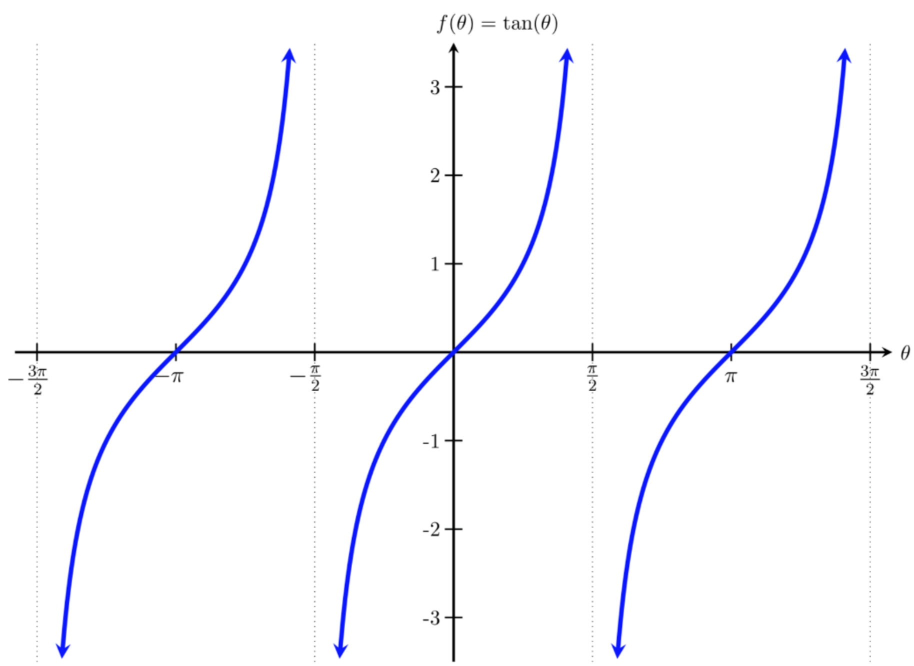


Figure 1: https://mathbooks.unl.edu/PreCalculus/tangent-and-cofunctions.html

Graphing the Trigonometric Function Worksheet Name: Group 3

Write your given function: y = sec(x)

1. What is the domain and range of your function?

   Domain: all real numbers except for points (2n + 1)π/2

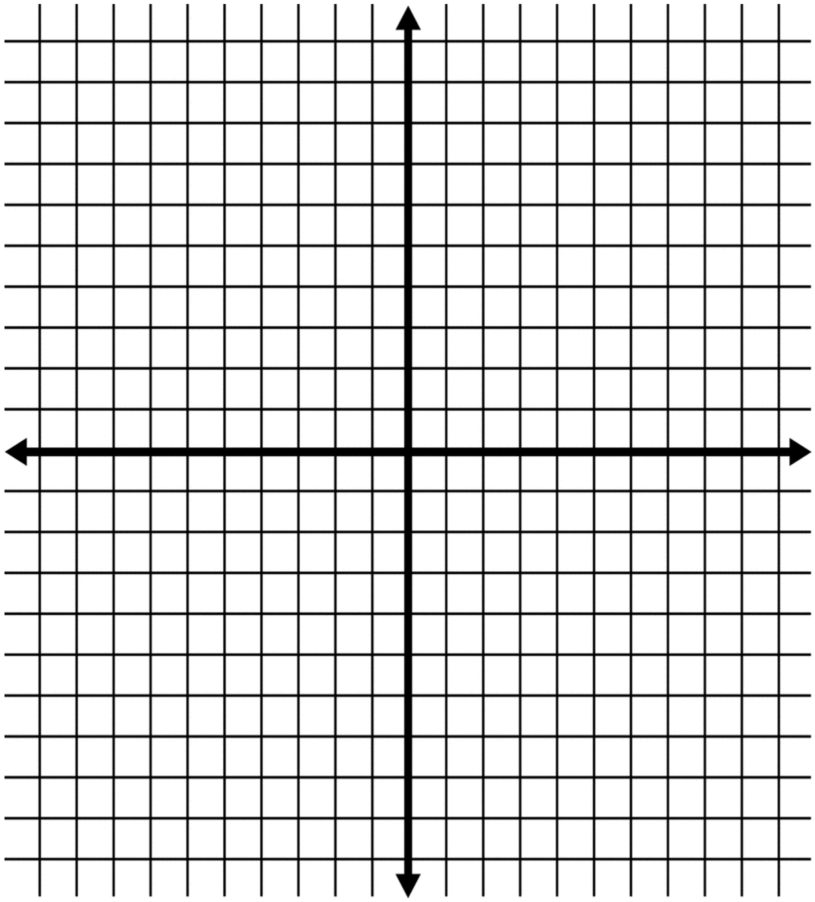
   Range: (-∞,-1] U [1, ∞)

2. If there are values not in your domain, what happens when you plug in numbers close to that value?

   Answers may vary.

3. Using the unit circle, fill in the table below.
4. Use the table and the information from number 3 to graph your function.

x	y
0	0
π/6	$\frac{2}{\sqrt{3}}$
π/3	2
π/2	Not defined
2π/3	-2
5π/6	$-\frac{2}{\sqrt{3}}$
π	-1
5π/4	$-\sqrt{2}$
3π/2	Not defined
7π/4	$\sqrt{2}$
2π	1



Look at the image below.

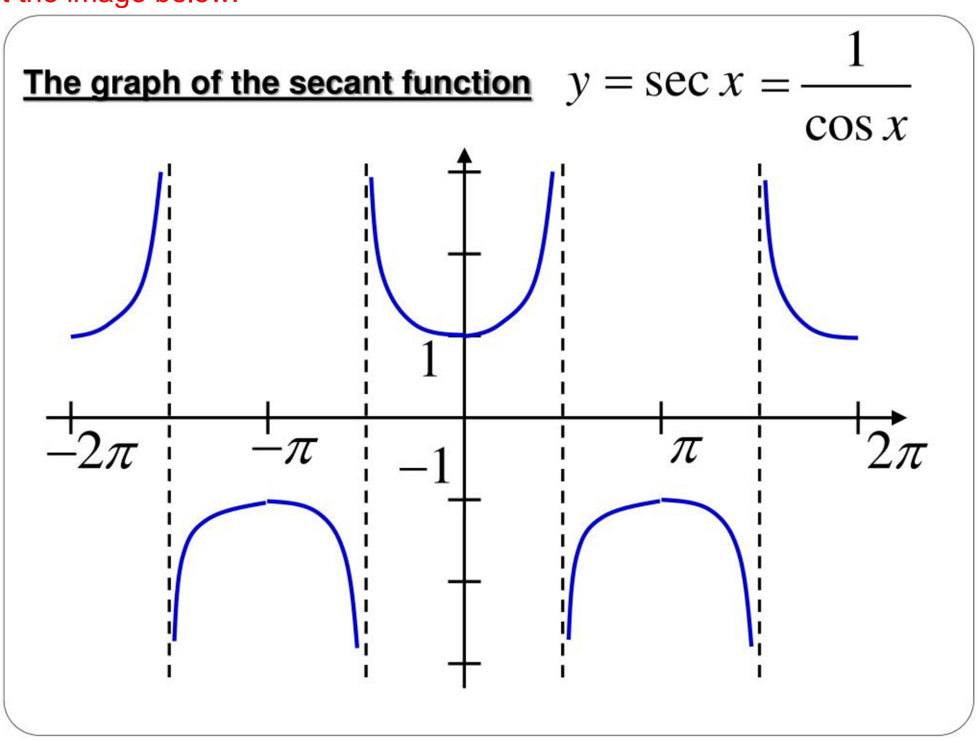


Figure 1; https://www.slideserve.com/drew/graphs-of-secant-and-cosecant-powerpoint-ppt-presentation

Graphing the Trigonometric Function Worksheet Name: Group 4

Write your given function: y = csc(x)

1. What is the domain and range of your function?

   Domain: all real numbers except nπ

   Range: (-∞, -1] U [+1, +∞)

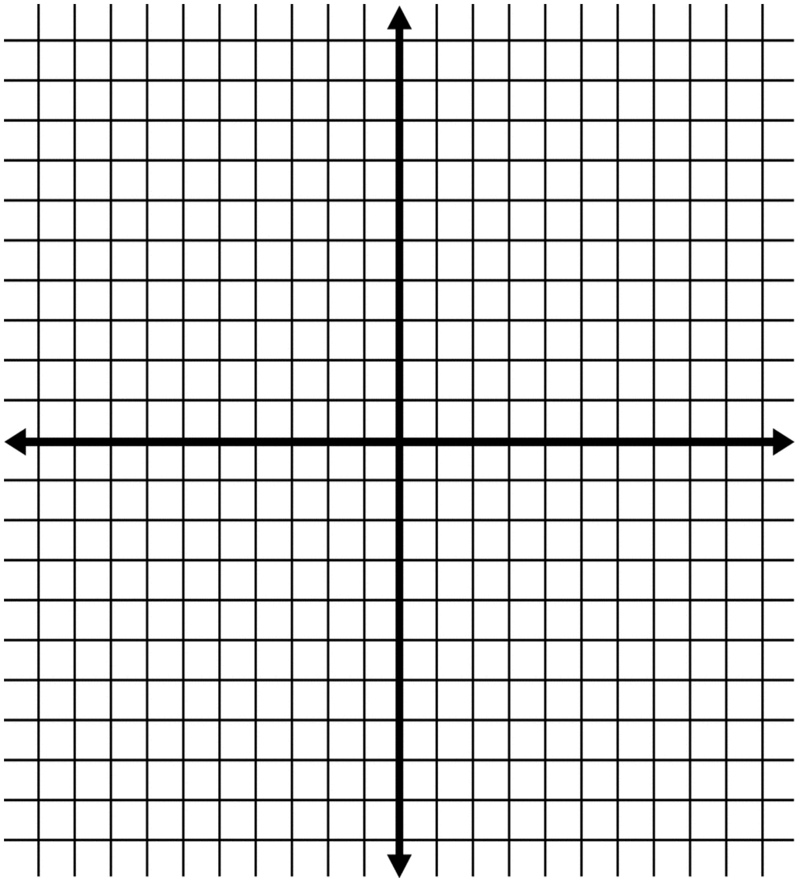
2. If there are values not in your domain, what happens when you plug in numbers close to that value?

   Answers may vary.

3. Using the unit circle, fill in the table below.

4. Use the table and the information from number 3 to graph your function.

x	y
0	Not defined
π/6	2
π/3	$\frac{2}{\sqrt{3}}$
π/2	1
2π/3	$\frac{2\sqrt{3}}{3}$
5π/6	2
π	Not defined
5π/4	$-\sqrt{2}$
3π/2	-1
7π/4	$-\sqrt{2}$
2π	Not defined



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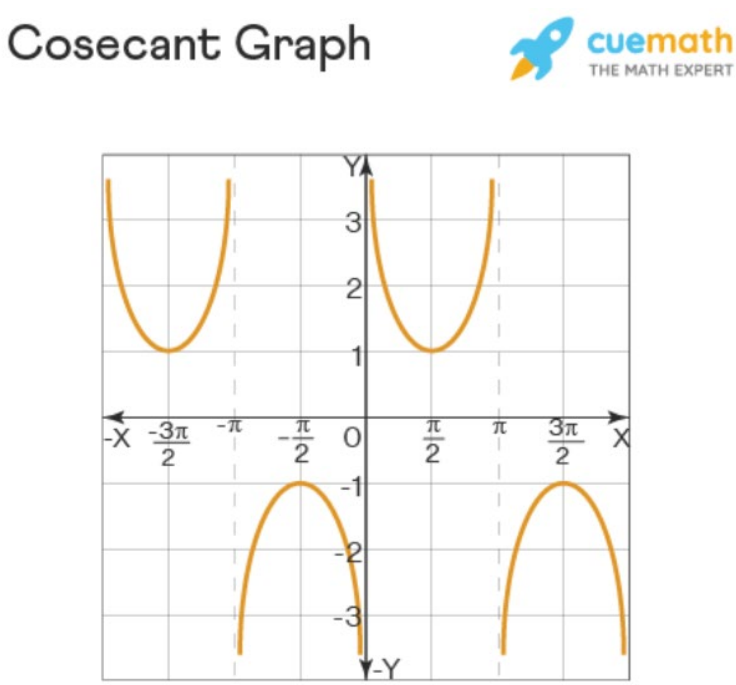


Figure 1: https://www.cuemath.com/trigonometry/cosecant-functions/

Graphing the Trigonometric Function Worksheet Name: Group 5

Write your given function: y = cot(x)

1. What is the domain and range of your function?

   Domain: all real numbers except nπ

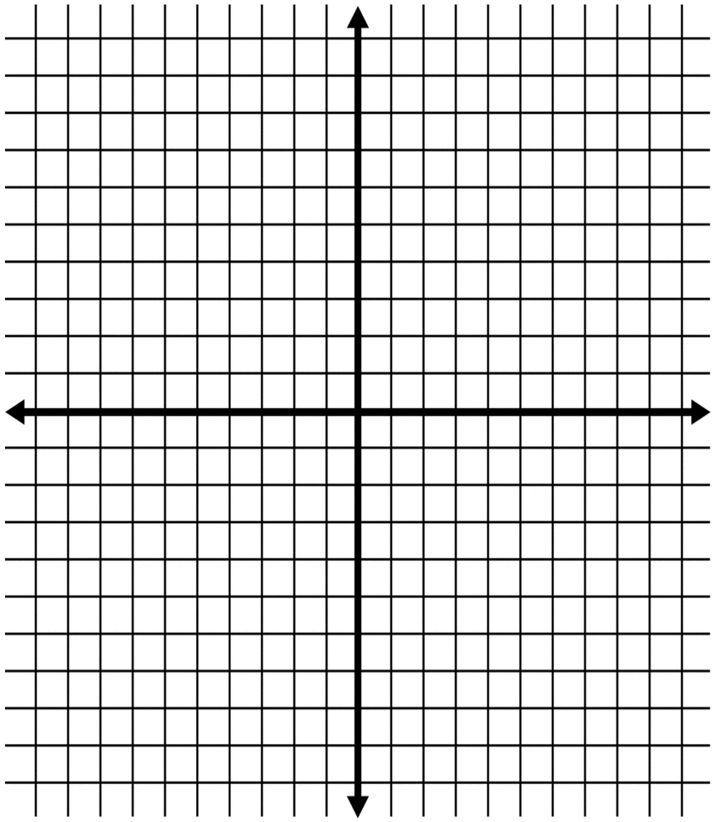
   Range: all real numbers

2. If there are values not in your domain, what happens when you plug in numbers close to that value?

   Answers may vary.

3. Using the unit circle, fill in the table below.
4. Use the table and the information from number 3 to graph your function.

x	y
0	undefined
π/6	$\sqrt{3}$
π/3	$\frac{\sqrt{3}}{3}$
π/2	0
2π/3	$-\frac{\sqrt{3}}{3}$
5π/6	$-\sqrt{3}$
π	undefined
5π/4	1
3π/2	0
7π/4	-1
2π	undefined



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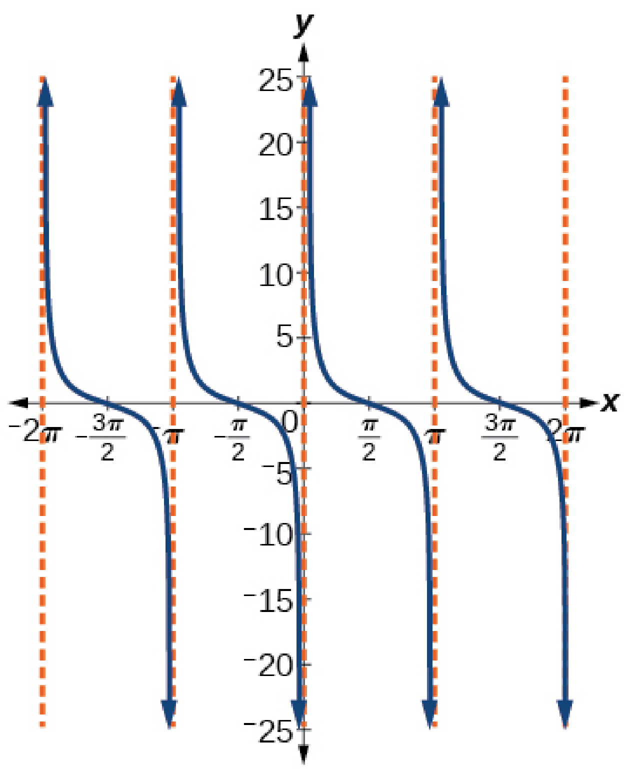
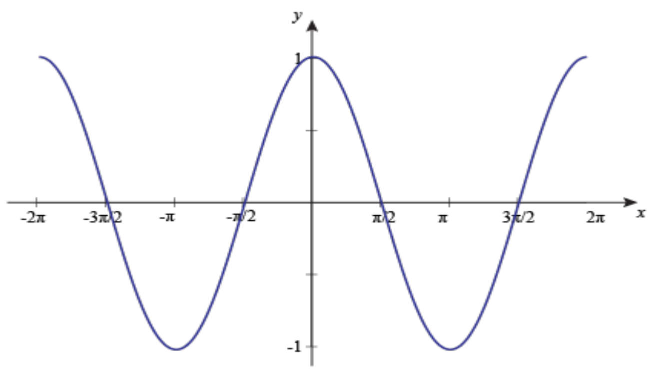


Figure 1: https://jobilize.com/trigonometry/test/graphing-variations-of-y-cot-x-by-openstax

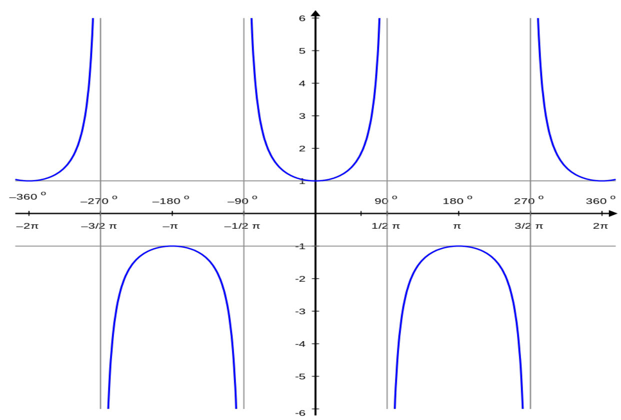
Graphing the Trigonometric Function Exit Ticket   Name:

Match the graphs with the corresponding function.

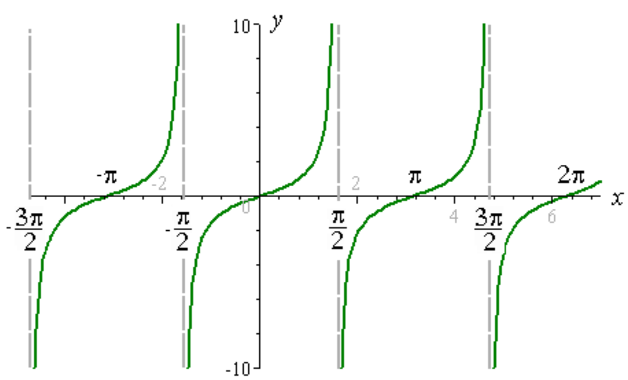
1. ___



2. ___



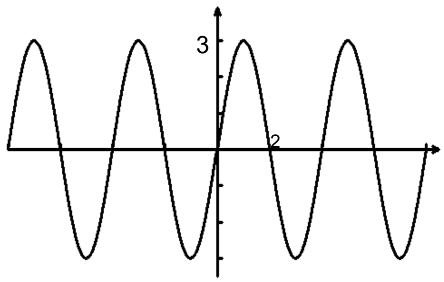
3. ___

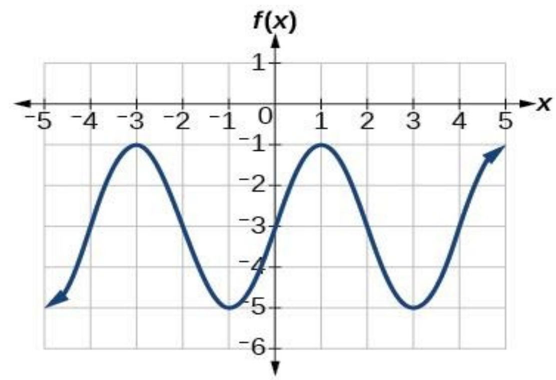


a. sin(x)
b. cos(x)
c. csc(x)
d. sec(x)
e. cot(x)
f. tan(x)

Identify the period and amplitude of each of graph.

4.





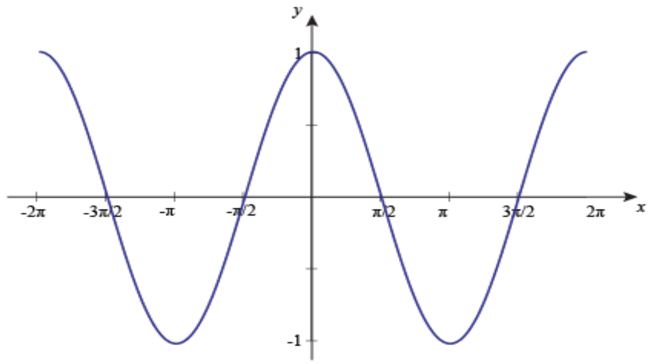
6. How does the domain of y = tan(x) relate to the asymptotes on the graph of this function?

7. How does the period of y = cos(x) relate to the unit circle?

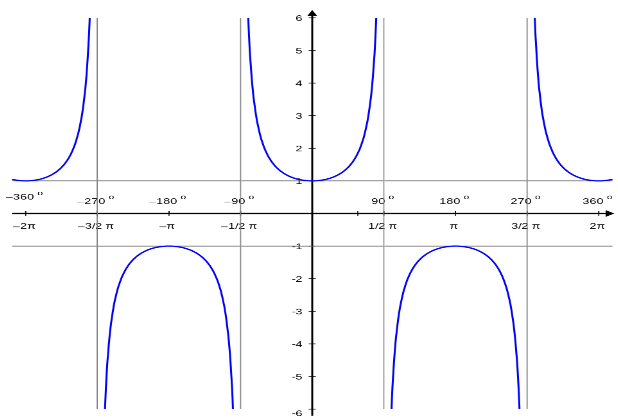
Graphing the Trigonometric Function Exit Ticket Name: KEY

Match the graphs with the corresponding function.

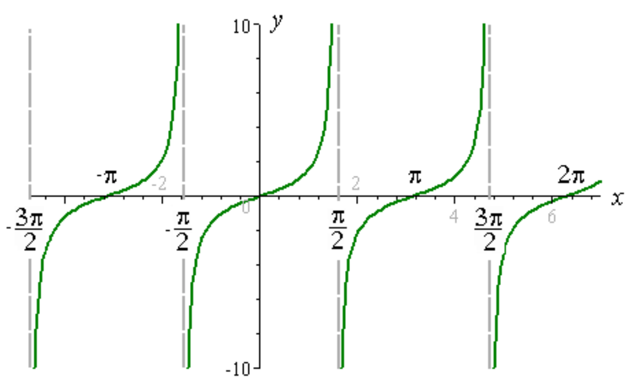
1. _b_



2. _d_



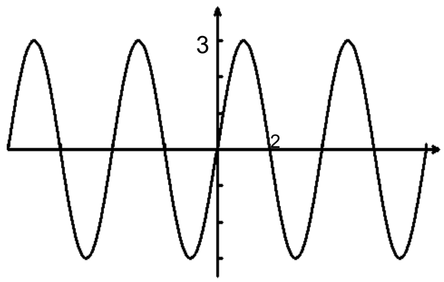
3. _f_



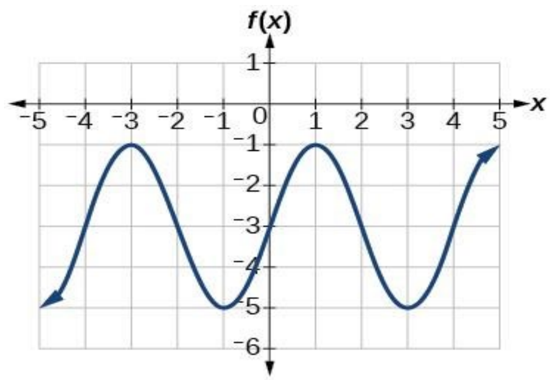
a. sin(x)
b. cos(x)
c. csc(x)
d. sec(x)
e. cot(x)
f. tan(x)

Identify the period and amplitude of each of graph.

4.



Amplitude: 3
Period: 4



Amplitude: 2
Period: 4

6. How does the domain of y = tan(x) relate to the asymptotes on the graph of this function?

Answers may vary.

7. How does the period of y = cos(x) relate to the unit circle?

Answers may vary.