Introduction to Proofs

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Target Grade: Geometry

Time Required: 75 minutes

Standards

Common Core Math Standards

- CCSS.MATH.CONTENT.HSG.CO.C.9
  Prove theorems about lines and angles.

Lesson Objectives

Students will:

- Solve Equations and justify solutions using a 2-column proof.

Central Focus

The lesson has an activity that uses the game of Uno to introduce proofs. In Uno, there are rules you must follow. These rules can be used to justify certain moves. Thus, students will be engaged with proofs in the form of a game to engage their attention. Writing proofs allows students to practice their logic skills. Logic is used across domains and is necessary for everyday functioning. Students will learn how to write proofs, which will help them organize their thinking and understand how to justify what they are doing.

Key Terms: proofs, addition property, distributive property, subtraction property, multiplication property, division property, substitution property, transitive property, 2-column proofs

Background Information

This lesson builds on the students' prior knowledge of how to solve equations. The students will be writing proofs that justify each step of solving an equation. To justify their steps, the students will use previously learned properties, such as the additive property, distributive property, and the multiplicative property.

Prior to this lesson, teachers should be familiar with the terms: addition property, distributive property, subtraction property, multiplication property, division property, substitution property and transitive property.

- Addition property
    - If a = b and c = d, then a + c = b + d.

- Distributive property
    - a(b + c) = ab + ac

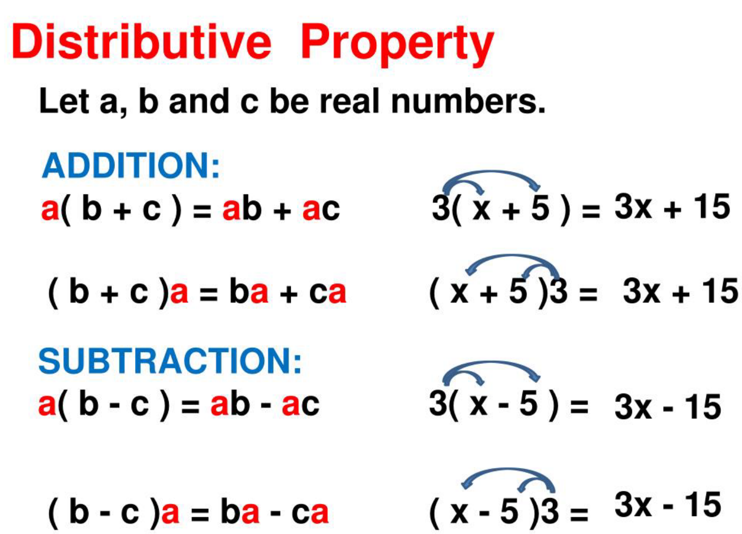
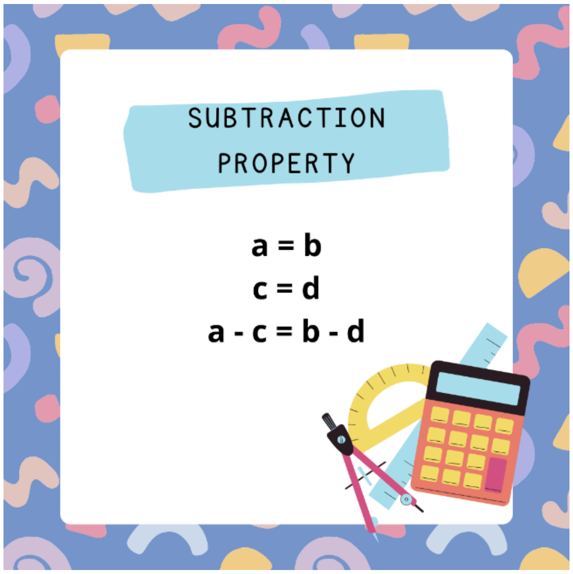
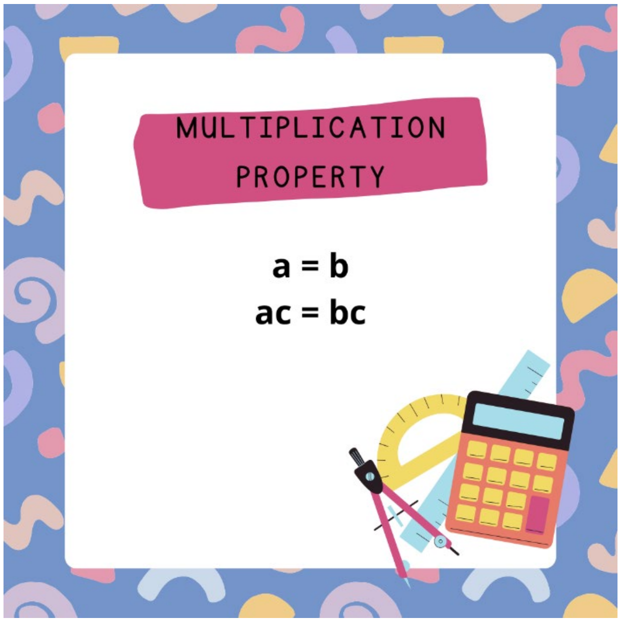


Figure 1: https://www.slideserve.com/ellema/1-7-distributive-property

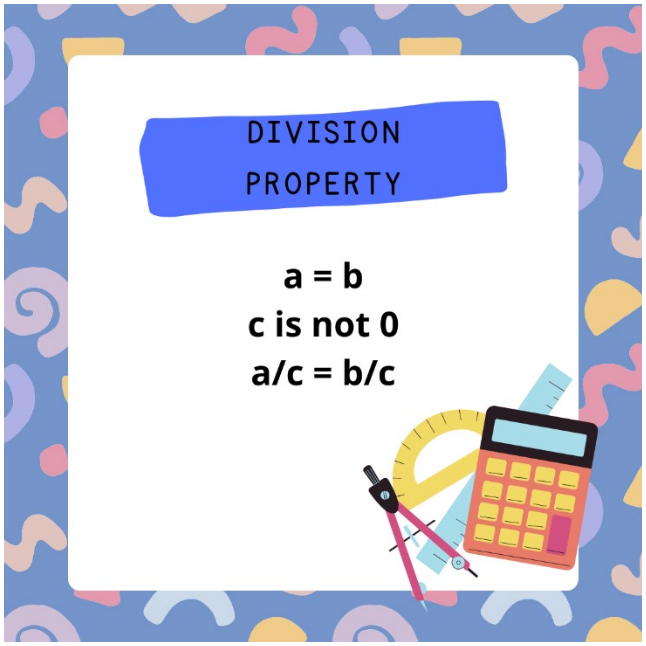
- Subtraction property
  - If a = b and c = d, then a - c = b – d.



- Multiplication property
  - If a = b, then ac = bc.



- Division property
  - If a = b and c is not 0, then a/c = b/c.



- Substitution property
  - If a = b, then a can be substituted for b in any equation or inequality.

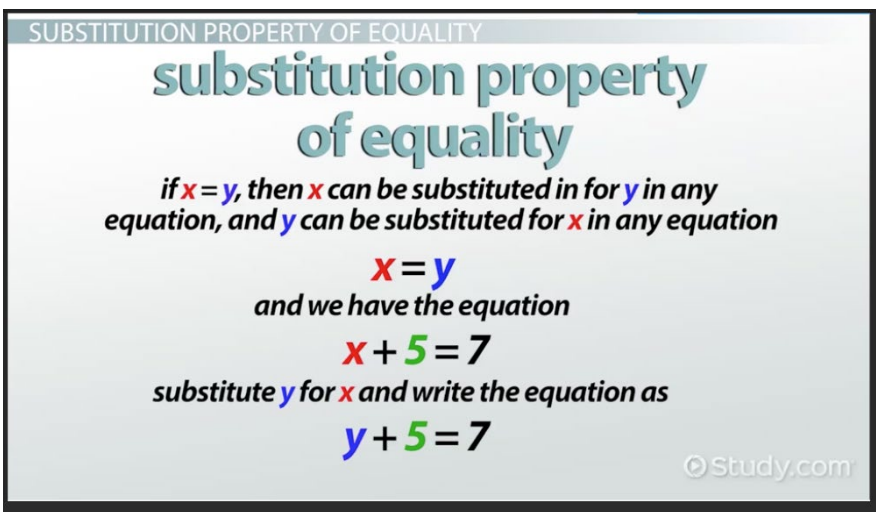


Figure 2: http://study.com/academy/lesson/substitution-property-of-equality-definition-examples.html?seekTo=%7B%7Bquiz.questionContent(questionIndex).marker%7D%7D

- Transitive property
  - If a = b and b = c, then a = c.

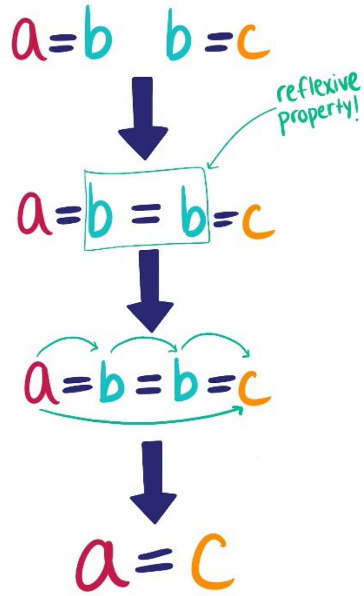


Figure 3: https://www.expii.com/t/transitive-property-of-equality-4155

Materials

- Whiteboard
- Uno Proofs Presentation (taken from MathTeacherCoach.com)
- Proofs Worksheet
- Proofs Worksheet Key
- Exit Ticket
- Exit Ticket Key

Instruction

Introduction (15 min)

- As a warmup, the students will answer the question, "What are the rules of Uno?"

Figure 4: https://www.mix1023.com.au/lifestyle/uno-steps-in-after-debate-sparks-over-controversial-rule-in-the-popular-card-game/

- Facilitate a discussion in which the class will create a list of the rules of Uno.
- Then, have the class complete the Uno proof activity (the teacher will give out the worksheets for the day).
- First, introduce the idea of a two-column proof, showing them a T-chart, with the statements (or "steps") on the left and the justifications for each step on the right.
- Then, as a class look at an example of using the columns to "prove" a sequence of Uno cards.
- Next, instruct the students to work by themselves on the next two Uno proof problems.

Transition

- Introduce the activity to the students by saying: "Just like there are rules in Uno that we can use to prove our steps, there are rules in algebra to explore these Uno rules."

Activity (50 min)

- Ask the students to solve the following equation: $4(x-1)+10 = 7x + 3$ to show that $x = 1$.
- Choose one student to show their work on the board.
- Next, show a table of algebraic properties, saying that these are the rules of algebra.
- Do a Think-Pair-Share to identify which property was used in each step of the equation.
- Call on students to share what they thought the justification of the steps were based on the properties.

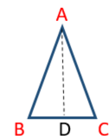
- As the students give their answers, write out the steps and justifications in a 2-column format.

Two Column Proof: Isosceles Triangle Theorem

The Isosceles Triangle Theorem states that if two sides of a triangle are congruent, then the angles opposite the sides are congruent.

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$



Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. Construct a bisector of $\angle A$	2. Every $\angle$ has 1 $\angle$ bisector
3. $\angle BAD \cong \angle CAD$	3. Definition of $\angle$ bisector
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\Delta ABD \cong \Delta ACD$	5. SAS Postulate
6. $\angle B \cong \angle C$	6. CPCTC

Figure 5: https://www.onlinemathlearning.com/two-column-proofs.html

- Put the students into groups of 3-4.
- Direct the students to work the next problems on the worksheet in groups.
  - In each group, there will be one manager (to make sure the group is on task), one fact-checker (to make sure what the group is doing is accurate), one reporter (to share the group's ideas to the class), and for groups of 4, one facilitator (to facilitate discussion between the group and make sure everyone understands the content).
- As the students work, monitor their discussions and select students to present their work during a whole class discussion.
- Next, have the class extend their learning with proofs to prove statements about geometrical objects.
- Begin by asking what types of information could be used as justification in geometry.
- Discuss how the properties of algebra, any definitions, and any postulates can be used.
- Instruct the students to begin filling out the proofs, and the teacher will select students to share their answers when the groups have had several minutes to work.

Closure (10 min)

- Hand out the exit ticket.
- End with an exit ticket on which students complete an algebraic proof and fill in missing steps of a geometric proof.
  - On the exit ticket, students have an opportunity to reflect on what they understood well and what they did not understand well during the lesson.

Differentiation

ELL Students

- Make a copy of the properties table in their language.
- Allow students to use an online dictionary.

For Students Who Need More Support

- Provide a “word” bank of statements and reasons they can choose from to write or fill in proofs.
- Provide a table of definitions and postulates that we have covered so far for the students to reference when choosing reasons in their proofs.

Grouping

- Students will be arranged in groups of 3-4 students.
- These groups will be heterogeneous to allow students to interact with their peers who have differing levels of knowledge and skills.
- In each group, there will be one manager (to make sure the group is on task), one fact-checker (to make sure what the group is doing is accurate), one reporter (to share the group’s ideas to the class), and for groups of 4, one facilitator (to facilitate discussion between the group and make sure everyone understands the content).

Advanced Students

- Allow students to move at a faster pace.
- Have the students help those who need assistance.
- Ask students to do a 2-column proof without any part filled out.

Assessment

Formative Assessment

- The class discussion will allow the teacher to see how much the students understood the concepts as well as what they may be struggling with.
- Monitoring the students during the activity can allow the teacher to hear what the students are understanding as well as what they may have questions with.

Summative Assessment

- The exit ticket assesses whether the students understand how to write an algebraic proof in the 2-column format and whether they can follow the logic of a geometric proof by filling in missing steps.

Sample

- Begin with 

- List how to play these cards to

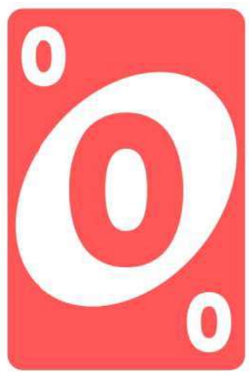
- ‘Prove’

T-Chart

Card played	Reason
1. Blue 6	1. Given
2. Blue Skip	2. Same Color
3. Wild Draw 4	3. Change Color
4. Yellow 5	4. Same Color
5. Yellow 1	5. Same Color
6. Yellow Reverse	6. Same Color

Given:

Prove: 

Using: 

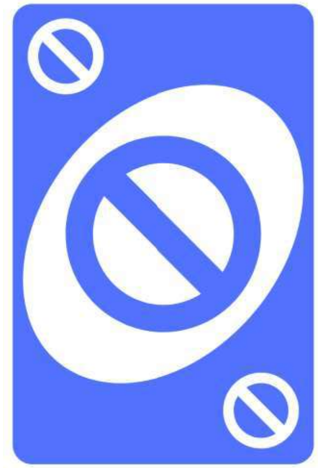
T-Chart

Given: Yellow 1

Prove: Red 2 

Statements	Reasons
1. Yellow 1	1. Given
2. Yellow 0	2. Same Color
3. Red 0	3. Same Number
4. Red 2	4. Same Color

Given: 

Prove: 

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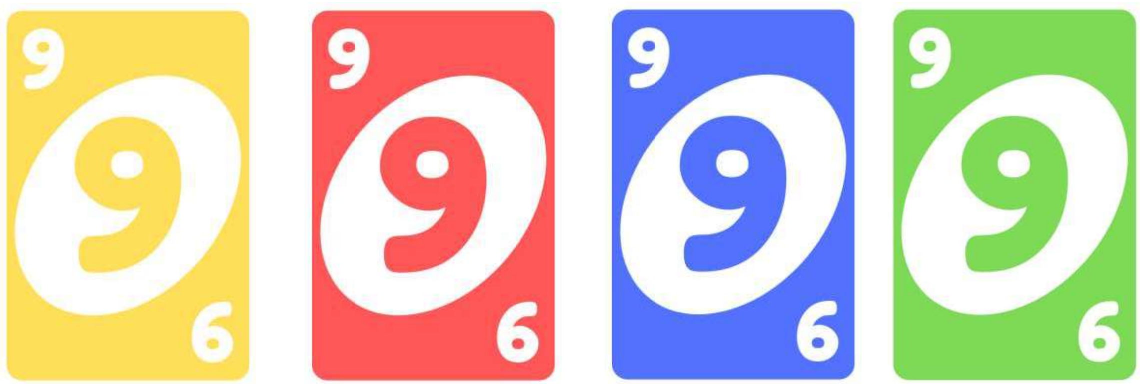
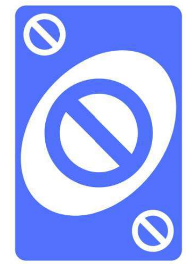


T-Chart

Given: Green Draw 2 

Prove: Blue Skip 

Statements *(What Card to Play)*:	Reasons *(I can play this card because)*:
1. Green Draw 2	1. Given
2. Green 9	2. Same Color
3. Blue 9	3. Same Number
4. Blue Skip	4. Same Color

The Red 9 does not have to be used. (It is ok to do so, but would require an additional step.)

Given: 

Prove: 

Using:

T-Chart

Given: Blue 9 

Prove: Red 2 

Statements:	Reasons:
1. Blue 9	1. Given
2. Blue Draw 2	2. Same Color
3. Green Draw 2	3. Same Symbol
4. Green 6	4. Same Color
5. Red 6	5. Same Number
6. Red 2	6. Same Color

Not all of the cards were used.

Prove It!

Let’s play uno!

Start: Yellow 1

End: Red

Steps	Justification

Start: Green +2

End: Blue skip

Steps	Justification

Start: Blue 9

End: Red 2

Steps	Justification

Let's go back to algebra:

Solve for x if 4(x-1) + 10 = 7x + 3.

Go back to your solution. Next to each step, write out which property you used.

Algebraic Properties
Addition Property: If a = b and c = d, then a + c = b + d.
Subtraction Property: If a = b and c = d, then a - c = b – d.
Multiplication Property: If a = b, then ac = bc.
Division Property: If a = b and c is not 0, then a/c = b/c.
Distributive Property: a(b + c) = ab + ac.
Transitive Property: If a = b and b = c, then a = c.
Substitution Property: If a = b, then a can be substituted for b in any equation or inequality.

1. Given: $\frac{x-1}{2} = 3x + 2$

Prove: x = -1

Statements	Reasons
1. $\frac{x-1}{2} = 3x + 2$	1. Given
2. $2(\frac{x-1}{2}) = 2(3x + 2)$	2.
3. $x - 1 = 2(3x + 2)$	3.
4. $x - 1 = 6x + 4$	4.
5. $x = 6x + 5$	5.
6. $-5x = 5$	6.
7. $x = -1$	7.

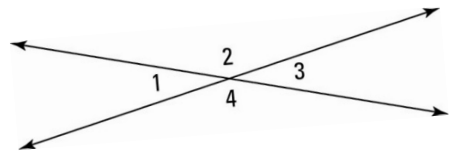
2. Given: $3x - 10 = 20 - \frac{1}{3}x$

Prove: x = 9

Statements	Reasons

What about proving geometrical statements? What other justifications could we use (in addition to our properties?)

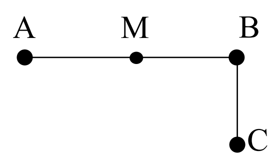
Try it out!



Given: $\angle 1$ and $\angle 3$ are vertical angles

Prove: $\angle 1 \cong \angle 3$

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 180^\circ$	1. Definition of supplementary angles
2. $m\angle 2 + m\angle 3 = 180^\circ$	2.
3.	3. Transitive Property
4. $m\angle 1 = m\angle 3$	4. Subtraction Property
5. $\angle 1 \cong \angle 3$	5.



Given: M is the midpoint of $\overline{AB}$

$\overline{MB} \cong \overline{BC}$

Prove: $\overline{AM} \cong \overline{BC}$

Statements	Reasons
1. M is the midpoint of $\overline{AB}$	1.
2.	2.
3. $\overline{MB} \cong \overline{BC}$	3.
4. $\overline{AM} \cong \overline{BC}$	4.

Prove It! Key

Let's play uno!

Answers will vary.

Start: Yellow 1

End: Red

Steps	Justification

Start: Green +2

End: Blue skip

Steps	Justification

Start: Blue 9

End: Red 2

Steps	Justification

Algebraic Properties
Addition Property: If a = b and c = d, then a + c = b + d.
Subtraction Property: If a = b and c = d, then a - c = b – d.
Multiplication Property: If a = b, then ac = bc.
Division Property: If a = b and c is not 0, then a/c = b/c.
Distributive Property: a(b + c) = ab + ac.
Transitive Property: If a = b and b = c, then a = c.
Substitution Property: If a = b, then a can be substituted for b in any equation or inequality.

Let's go back to algebra:

Solve for x if 4(x-1) + 10 = 7x + 3.

4(x-1) + 10 = 7x + 3 given

4x-4 + 10 = 7x + 3 distributive property

4x + 10 = 7x + 7 addition property

3 = 3x subtraction property

1 = x division property

Go back to your solution. Next to each step, write out which property you used.

Answers may vary depending on each step.

1. Given: $\frac{x-1}{2} = 3x + 2$

Prove: x = -1

Statements	Reasons
1. $\frac{x-1}{2} = 3x + 2$	1. Given
2. $2(\frac{x-1}{2}) = 2(3x+2)$	2. Multiplication property
3. $x - 1 = 2(3x+2)$	3. Substitution property
4. $x - 1 = 6x + 4$	4. Distributive property
5. $x = 6x + 5$	5. Addition property
6. $-5x = 5$	6. Subtraction property
7. $x = -1$	7. Division property

2. Given: $3x - 10 = 20 - \frac{1}{3}x$

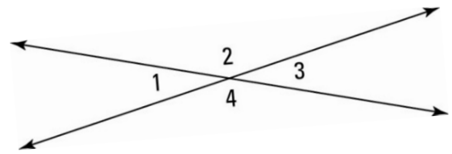
Prove: x = 9

Statements	Reasons
1. 3x – 10 = 20 - $\frac{1}{3}x$	1. Given
2. 3(3x – 10) = 3(20 – $\frac{1}{3}x$)	2. Multiplication property
3. 9x – 30 = 60 – x	3. Distributive property
4. 9x = 90 – x	4. Addition property
5. 10x = 90	5. Addition property
6. X = 9	6. Division property

What about proving geometrical statements? What other justifications could we use (in addition to our properties?)

Answers may vary.

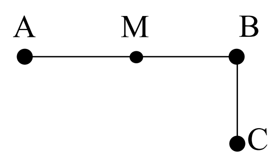
Try it out!



Given: ∠1 and ∠3 are vertical angles

Prove: ∠1 ≅ ∠3

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 180^{\circ}$	1. Definition of supplementary angles
2. $m\angle 3 + m\angle 2 = 180^{\circ}$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property
4. $m\angle 1 = m\angle 3$	4. Subtraction Property
5. $\angle 1 \cong \angle 3$	5. Definition of congruence



Given: M is the midpoint of $\overline{AB}$

$\overline{MB} \cong \overline{BC}$

Prove: $\overline{AM} \cong \overline{BC}$

Statements	Reasons
1. M is the midpoint of $\overline{AB}$	1. Given
2. $\overline{AB} \cong \overline{MB}$	2. Definition of midpoint
3. $\overline{MB} \cong \overline{BC}$	3. Given
4. $\overline{AM} \cong \overline{BC}$	4. Transitive property

Exit Ticket

Name: ___________________________

1. Identify the property that justifies each step.

$2x + 6 = 4(x + 1)$ Given ________________

$2x + 6 = 4x + 4$ ________________________

$2x + 2 = 4x$ ________________________

$2 = 2x$ ________________________

$1 = x$ ________________________

2. Fill out the table with the missing statements and reasons.

Given: $\angle 1$ and $\angle 2$ are complementary, $m\angle 1 = 54^\circ$

Prove: $m\angle 2 = 36^\circ$

Statements	Reasons
	Given
$m\angle 1 + m\angle 2 = 90^\circ$	
$m\angle 1 = 54^\circ$	
	Substitution Property
$m\angle 2 = 36^\circ$	

3. What did you find most difficult about this lesson?

4. What is something you understand well from this lesson?

Exit Ticket KEY

Name: ___________________________

1. Identify the property that justifies each step.

$2x + 6 = 4(x + 1)$ Given

$2x + 6 = 4x + 4$ Distributive Property

$2x + 2 = 4x$ Subtraction property

$2 = 2x$ Subtraction property

$1 = x$ Division property

2. Fill out the table with the missing statements and reasons.

Given: $\angle 1$ and $\angle 2$ are complementary, $m\angle 1 = 54^\circ$

Prove: $m\angle 2 = 36^\circ$

Statements	Reasons
$\angle 1$ and $\angle 2$ are complementary	Given
$m\angle 1 + m\angle 2 = 90^\circ$	Definition of complementary
$m\angle 1 = 54^\circ$	Given
$54^\circ + m\angle 2 = 90^\circ$	Substitution Property
$m\angle 2 = 36^\circ$	Subtraction property

3. What did you find most difficult about this lesson?

Answers will vary.

4. What is something you understand well from this lesson?

Answers will vary.