Activity: Calculating Average Distance Biomass will Travel to a Biorefinery

Grade Level: 12th grade Calculus

Standards

- **TN state standard: C.I.AI.B.5** Use a definite integral to find the volume of a solid formed by rotating a region around a given axis.
- **AP calculus standard: LO 3.4D** Apply definite integrals to problems involving area and volume.

Learning Objectives:

- Find a one year production density
- Write and evaluate an integral to find the volume of a solid in a real life context where coordinate plane and equation are not provided.
- Use calculus to create a generalized formula for a real life calculation.

Materials: None

Time Required:

Background Information: Bioenergy is energy derived from living matter on the surface of the earth. In the United States, we are at a historic high of renewable energy use. Bioenergy is the highest source of renewable energy. Biomass, the raw source of bioenergy, includes forests, agriculture, trash and algae. Like an oil refinery takes crude oil and converts it to a form that can be used commercially, a biorefinery takes biomass and converts it to a usable form by the consumer.

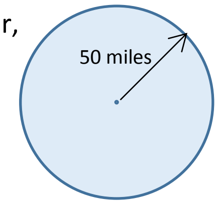
Focusing on agriculture, there are factors to consider when deciding if a biorefinery location will be viable. One consideration is how much material can be grown near the biorefinery and another is transportation costs. Transportation costs depend on the average distance material needs to travel in order to reach the biorefinery.

Below is a flow chart that shows the general steps that are taken in creating biofuel from agriculture. The following problem focuses on the two bolded sections.

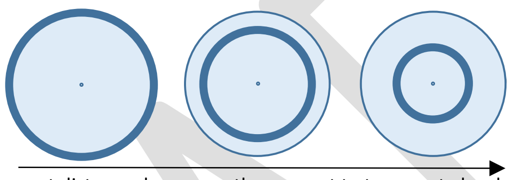
Name:_______________________________________ Date:_______________ Period:___________

Biofuel Application: The specifications for a planned biorefinery require 800,000 dry tons of biomass per year. The designers expect that all the biomass will come from a 50 mile radius from the biorefinery.

1) What is the biomass production density of material produced in one year, in dry tons per square miles? Round to the nearest thousandth.



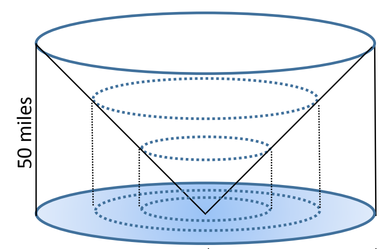
To calculate cost to transport the biomass to the biorefinery the average distance that biomass is transported is needed. If you view fields as concentric circles, you can see that the outermost ring has the most biomass (biggest circumference). As the circles get closer to the center, the location of the biorefinery, the distance decreases, but so does the amount of biomass that needs to be transported.



As transport distance decreases, the amount to transport also decreases.

To calculate the average distance (not just half of the radius!) that biomass is transported, a 3D model is employed. Initially the volume of the solid is calculated, in miles^3.

2) Write an integral expression to represent the volume above the circle and below the cone.



3) Evaluate the integral.

4) To find average distance the volume found in the last problem is divided by the area of base (shaded above). Calculate the average distance and include units.

5) Write and evaluate an integral expression to find the average transport distance for a circle of any radius, r, around a biorefinery.

Answer Key

Biofuel Application: The specifications for a planned biorefinery require 800,000 dry tons of biomass per year. The designers expect that all the biomass will come from a 50 mile radius from the biorefinery.

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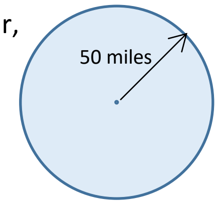
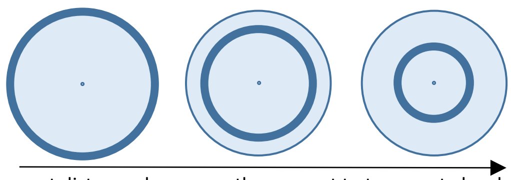


$$\frac{800{,}000}{\pi(50)^2} = \frac{320}{\pi} \approx 101.859 \ \frac{\text{dry tons}}{\text{mi}^2}$$

To calculate cost to transport the biomass to the biorefinery the average distance that biomass is transported is needed. If you view fields as concentric circles, you can see that the outermost ring has the most biomass (biggest circumference). As the circles get closer to the center, the location of the biorefinery, the distance decreases, but so does the amount of biomass that needs to be transported.



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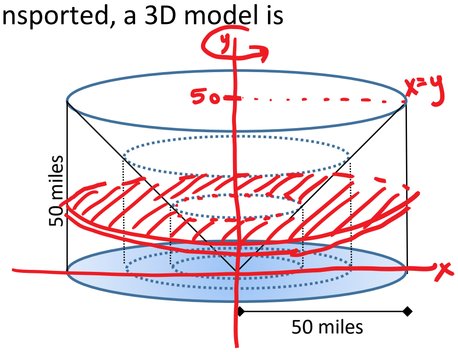
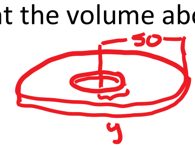


2) Write an integral expression to represent the volume above the circle and below the cone.

$$\pi\int_0^{50}(50^2 - y^2)\,dy$$



3) Evaluate the integral.

$$\pi\int_0^{50}(50^2 - y^2)\,dy$$

$$\pi\left[50^2 y - \tfrac{1}{3}y^3\right]_0^{50}$$

$$\pi\left[\left(50^3 - \tfrac{1}{3}50^3\right) - 0\right] = \tfrac{2}{3}\pi(50)^3$$

4) To find average distance the volume found in the last problem is divided by the area of base (shaded above). Calculate the average distance and include units.

$$\frac{\frac{2}{3}\pi(50)^3}{\pi(50)^2} = \tfrac{2}{3}(50) = \frac{100}{3} \text{ mi}$$

5) Write and evaluate an integral expression to find the average transport distance for a circle of any radius, r, around a biorefinery.

$$\text{Ave. distance} = \frac{\pi\int_0^r (r^2 - y^2)\,dy}{\pi r^2}$$

$$= \frac{1}{r^2}\int_0^r (r^2 - y^2)\,dy$$

$$= \frac{1}{r^2}\left[r^2 y - \tfrac{1}{3}y^3\right]_0^r$$

$$= \frac{1}{r^2}\left[\left(r^2 \cdot r - \tfrac{1}{3}r^3\right) - 0\right]$$

$$= \frac{1}{r^2}\left[r^3 - \tfrac{1}{3}r^3\right]$$

$$= \frac{1}{r^2}\cdot\frac{2r^3}{3}$$

$$\boxed{\text{Average distance} = \tfrac{2}{3}r}$$