Area and Perimeter

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Target Grade: Geometry

Time Required: 75 minutes

Standards

Common Core Math Standards:

- CCSS.MATH.CONTENT.HSG.GPE.B.7
  Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., for example, using the distance formula.

Lesson Objectives

Students will:

- Calculate the perimeter and area of a polygon in the coordinate plane using the distance formula.
- Identify the measurements needed to calculate the perimeter and area of a polygon.

Central Focus

In this lesson, students will find the area and perimeter of the front of the greenhouses at their school in order to fit insulation on the greenhouse. The students will also engage in an activity about making improvements to a soccer field. By using an example that is relevant to the real world, students will be able to connect it to their personal experiences. Incorporating a problem that has students applying their knowledge to solve a real-world problem will help them engage with the mathematical concepts as well as practical thinking.

Key Words: distance, perimeter, distance equation, area, Pythagorean Theorem

Background Information

Prior to this lesson students will need to be familiar with area, perimeter, and symbolic expressions to represent area and perimeter. This lesson uses the concept of area and perimeter and adds a twist that requires abstract thinking. Students will be required to use symbolic expressions to represent the area and perimeter of figures with specific reference to line segments, and they will need to identify pieces of information they need in order to calculate area.

Prior to this lesson the teacher will need to be familiar with the Pythagorean Theorem, distance formula, and area and perimeter formulas.

- Pythagorean Theorem:
  $a^2 + b^2 = c^2$ (Note: c = the hypotenuse)
- Distance Formula:
  $d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$
- Rectangle
  - Area Formula:
    $A = l * w$ (Note: l = length; w = width)
  - Perimeter Formula:
    $P = 2l + 2w$ (Note: l = length; w = width)

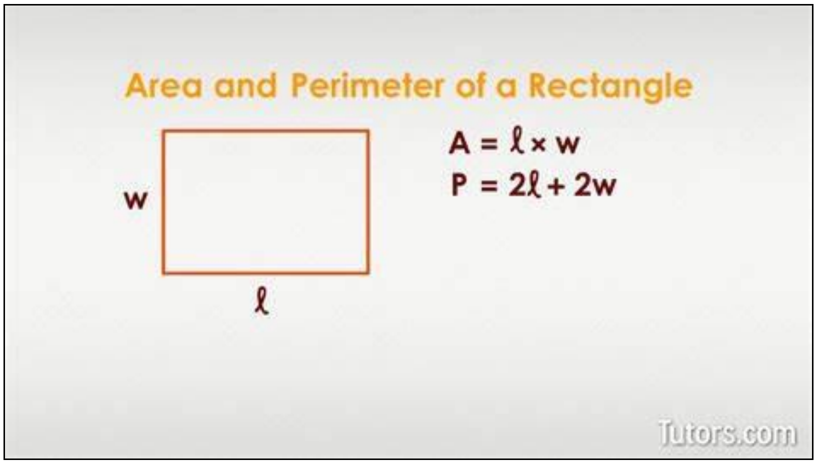


Figure 1: https://tutors.com/math-tutors/geometry-help/area-and-perimeter

- Triangle
  - Area Formula:
    $A = \frac{1}{2}bh$ (Note: b = base; h = height)
  - Perimeter Formula:
    $P = a + b + c$

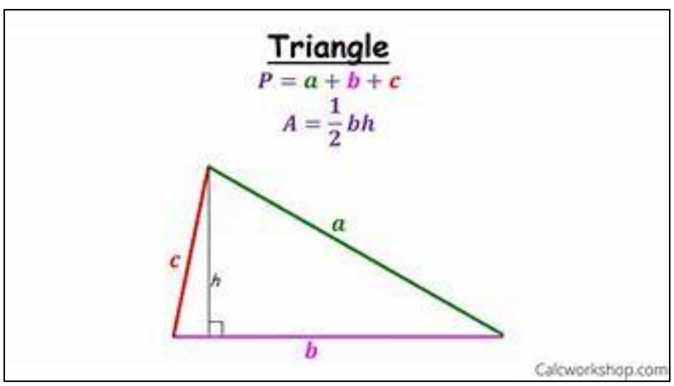


Figure 2: https://calcworkshop.com/basic-geometry/area-perimeter/

Materials

- Whiteboard
- TV (to display the presentation and notes)
- Digital drawing board (to write notes that will show up on the TV)
- Chromebooks, one per student
- Insulating the Greenhouses presentation
- Soccer Field Spruce Up worksheet
- Area & Perimeter on the Coordinate Plane (HW)
- Insulating the Greenhouses presentation Answer Key
- Soccer Field Spruce Up worksheet Answer Key
- Area & Perimeter on the Coordinate Plane (HW) Answer Key
- Notebook paper
- Pencils/pens
- Calculators

Instruction

Before Class Begins:

- Sketch polygons on the board with their vertices labeled. Do not give measurements given for the sides.
- Write instructions on the board that say, "Write an expression for the area and perimeter of the shape using the information given."

Warmup (10 min):

- Have the students follow the instructions on the board.
    - Students may be uncomfortable using symbols to write an expression for these values, instead of calculating them with numbers.
    - Allow students individual think time and time to discuss their ideas with the peers around them.
- Highlight several students' ideas and give support to students who do not know where to begin.
    - This activity helps activate their prior knowledge of calculating the area and perimeter of a rectangle and triangle, and it connects this knowledge to their recent learning of how to name line segments using points.
- During the warm up ask the following questions to deepen the students' learning:
    - What are the formulas for area and perimeter for these shapes?
    - How can you use our new vocabulary to describe the base (or length, or height)?
    - How can you use your knowledge of naming segments to rewrite the formulas for these specific shapes?
    - What can we do when we aren't explicitly given the measures of side lengths?

Greenhouse Activity (30 min):

- The teacher will launch into the "Insulating the Greenhouse" task, which incorporates whole class learning as well as small group collaboration (i.e. a "we do" style of learning).
    - This activity introduces students to calculating area and perimeter on the coordinate plane as students help their high school figure out how to cover their greenhouses with insulation.
- Students will be asked what information they need in order to solve the problem: The staff has already found the area and perimeter of the sides, but they are unsure of how to find the area of the front and back.
    - Students will have time to individually think about this question and talk to their peers about it.
- Then, the class will discuss how they need to find the area of the greenhouse to know how much insulation to buy and perimeter in order to properly install the insulation.
- Students will then be shown a diagram of the front of the greenhouse on a coordinate plane.

- Go over the key pieces of information the students need, such as the coordinates of the vertices. Key pieces of information:
    - Coordinates of vertices are needed in order to find the distance between two points. Finding the distance between the two points will allow the students to find the length of the sides.
    - Pythagorean Theorem is needed in order to find the length of some of the sides of the greenhouse.
    - The distance formula is needed in order to find the length of some of the sides of the greenhouse.
    - The triangle and rectangle area formula is needed to find the total area of the greenhouse.
- Then, students will work either individually or in a group in order to find the area and perimeter.

- During the Greenhouse Activity here are some questions the teacher may ask their students:
    - What information do we need to know in order to cover the greenhouse front with insulation?
    - What information do you need to find the perimeter of a triangle that is different from what you need to find its area?
    - Can you think of another way to find the area of the triangular part?
    - How can you find the area of a composite shape?
    - How can you calculate distances of a line segment when you only have information about its endpoints?

Figure 3: https://zionstar.net/ground-plan-maker/

Group work (25 min):

- Give students the "Soccer Field Spruce Up" worksheet to complete in small groups of 3-4 students.
    - This worksheet gives students an opportunity to apply their learning to a new situation and practice using the distance equation to find area and perimeter on the coordinate plane.
- Monitor the students as they work and be available to give support and answer questions.
- As the teacher monitors the students, here are some questions to ask the students:
    - Why is it difficult to solve for the area and perimeter of a rectangle in this problem?
    - Would you prefer to use the Pythagorean Theorem or the distance formula for this problem? (and why?)
    - What information do you know?
    - What information do you need to know?

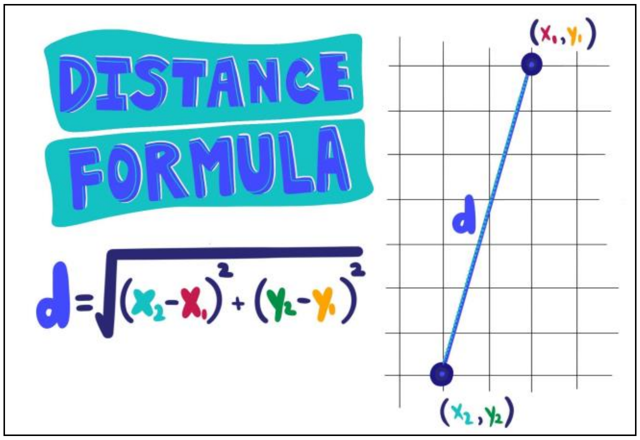


Figure 4: https://www.expii.com/t/distance-formula-4560

Summary (10 min):

- The class will reconvene for a discussion about any misconceptions that were identified during the group work and what students found difficult during the activities.
- Students will complete a short exit ticket in which they write down one thing they learned and one thing they are still confused about or find challenging.

Closure (5-10 minutes):

- The closure will involve a class discussion about common misconceptions among the students during the activities.
    - Remind students during this discussion that they may use either the Pythagorean Theorem or the distance formula to solve these problems, since they are synonymous.
- If time allows, ask students how their work in class made use of ideas they learned previously in the class.
- The teacher may give the students homework at the end of the lesson in order to assess their learning. (See homework attached below.)

Differentiation

Grouping:

- During the exploration activity and the guided practice, students will collaborate with 2-3 other students seated near them. These collaboration groups will be heterogeneous to allow students to support one another.
- Students will have worked with the peers in their group previously, so there will already be rapport among the group members.

ELL Students:

- Allowing dictionaries/use of online translation service
- Pairing students in heterogeneous groups for support
- Give extra time for students to solve problems
- Preferential seating for certain students
- Give written and oral instructions

Possible misconceptions:

- Students may think that simply naming the formula for the warmup will work and not realize that they need to be more specific. For example, using “a” to represent the base of a triangle works only if the triangle you are referring to has a base leg that is labelled “a”.
- Students may accidentally switch the x and y values when reading coordinates. When trying to find the length of a line that is not horizontal or vertical on the coordinate plane, some students may think this distance is simply the difference between the x values (or the y values).

Possible learning difficulties:

- Many students may not realize that they can use either the Pythagorean Theorem or the distance formula.
- Some students may have difficulty with the algebraic expressions, such as knowing how to square values or take their square roots. Students may have difficulty identifying what lengths they need, such as finding the correct segment for the height of a triangle.

Assessment

Formative Assessment:

- The teacher will check for understanding through classroom discussion. The teacher will help facilitate discussion and correct misconceptions, if necessary.
- The exit ticket at the end of the lesson will help gauge student understanding.
- The warmup will allow the teacher to check for understanding of the material as well as the end of class discussion about the activity.
- During the Greenhouse Activity, the teacher will ask the students questions which will allow the teacher to assess the students understanding.

Summative Assessment:

- The Soccer Field Spruce Up worksheet will allow the teacher to assess the students understanding of area and perimeter.
- The Area & Perimeter on the Coordinate Plane (HW) will allow the teacher to assess the students understanding of the lesson.

Insulating our Greenhouses

Keeping the Greenhouses Warm

Once the cold weather comes, our school is planning to cover its greenhouses with insulation to keep them warm.

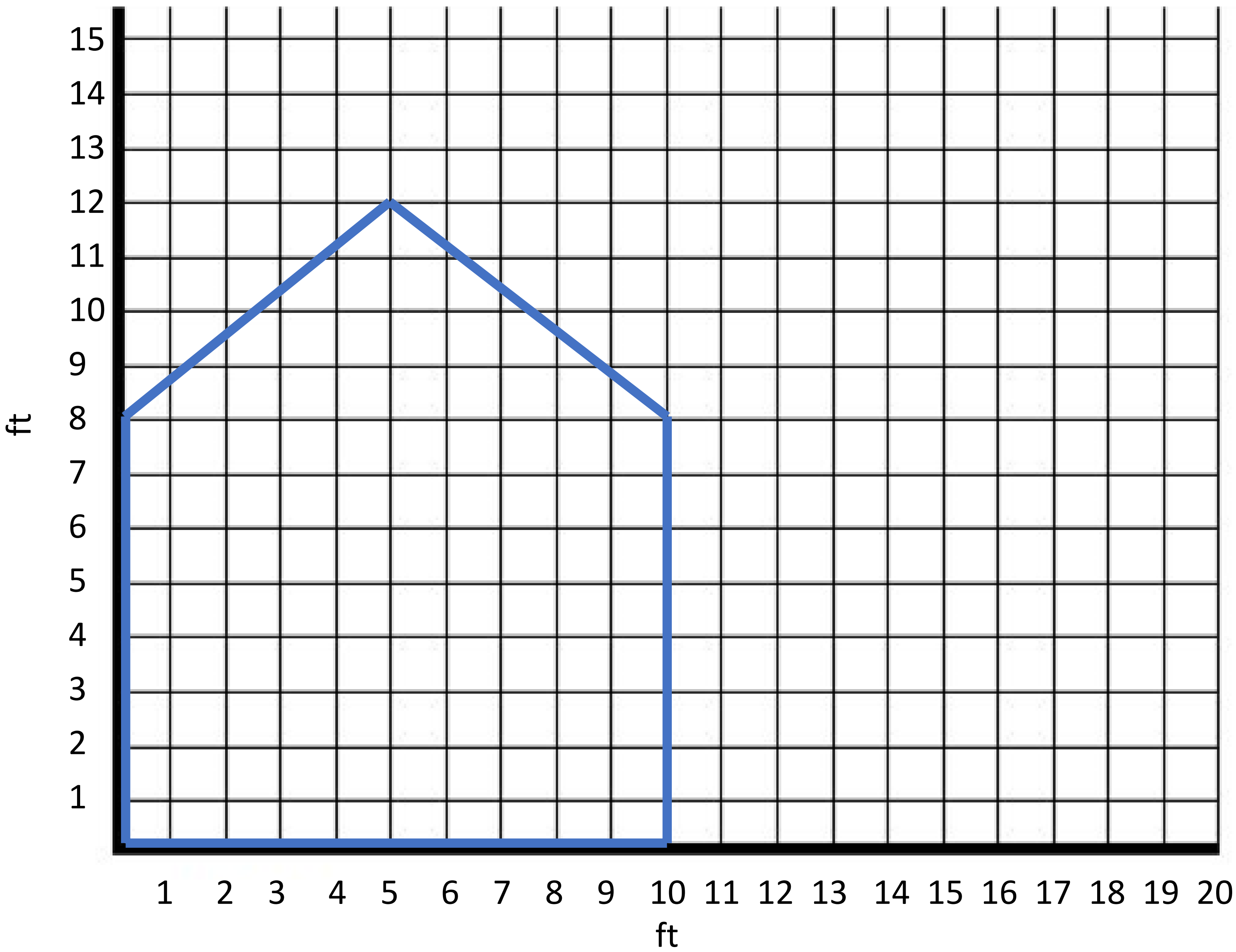
- What information do we need in order to buy the correct amount of insulation?
- What information do we need in order to cut the insulation out in the right shape?

The Problem

The staff has already found the area and perimeter of the sides, but they are unsure of how to find the area of the front and back.

Your Mission

Your school is relying on its geometry students to figure out the area and the perimeter of the front and back of the greenhouses. They have provided you with the following diagram to help…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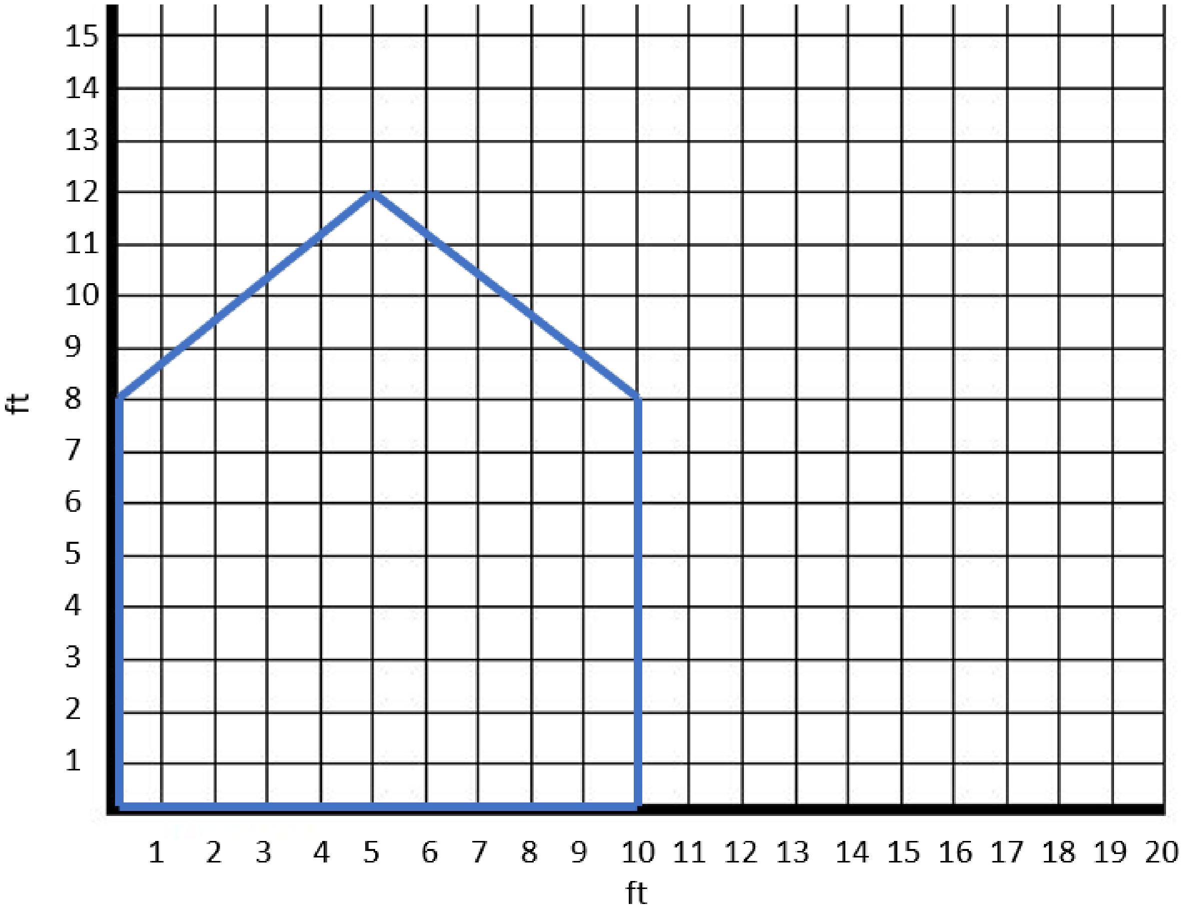

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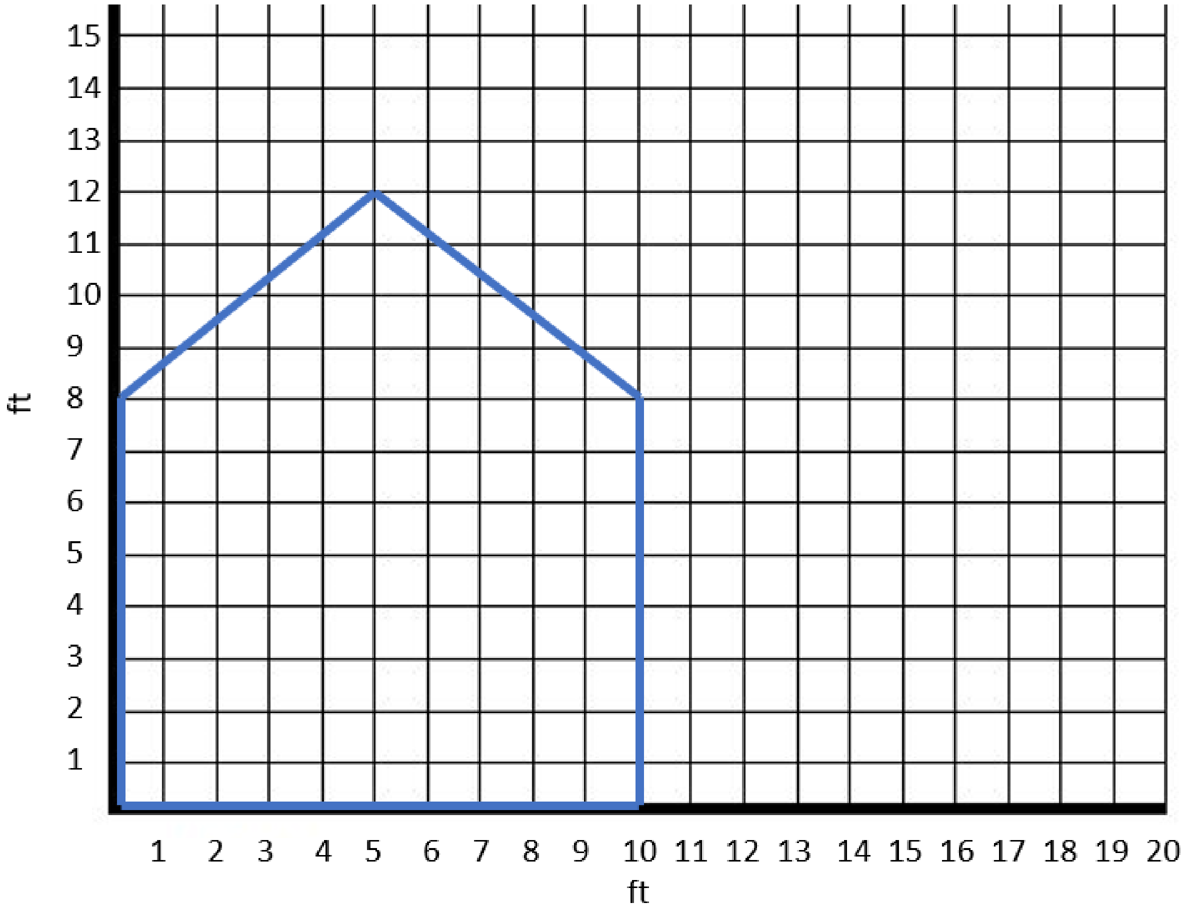

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Example Notes to write based on discussion with students

Distance Formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Rectangle

Area

$A = l \times w$

Perimeter

$P = 2l + 2w$

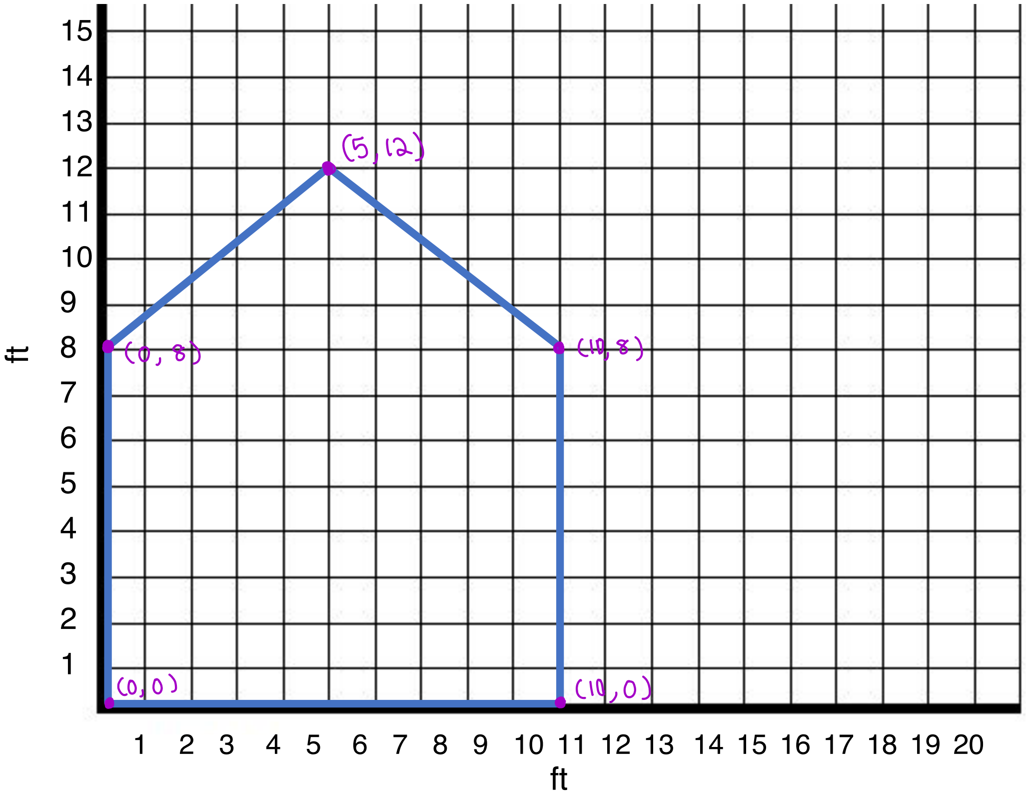
Triangle

Area

$A = \frac{1}{2}bh$

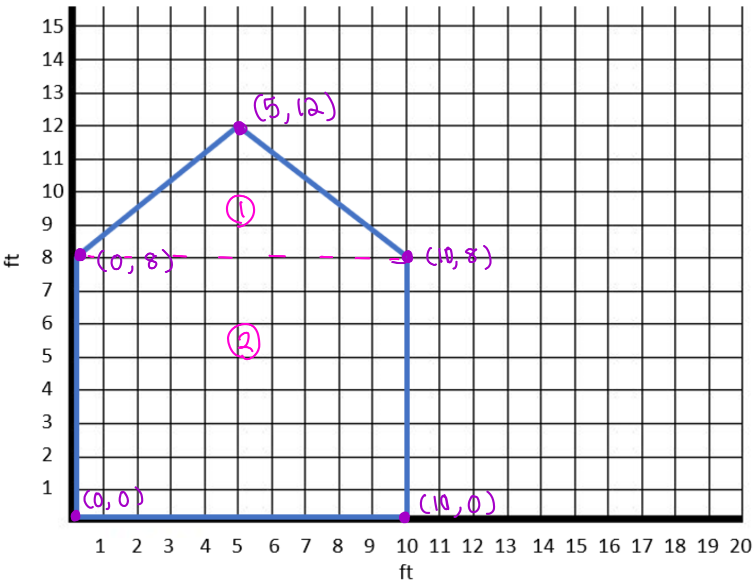
Perimeter

$P = a + b + c$



Key

NOTE: There are other ways to solve this problem.



Area

① $b = 10$ units $h = 4$ units

$A = \frac{1}{2}bh = \frac{1}{2}(10)(4) = 20 \text{ units}^2$

② $b = 10$ units $\ell = 8$ units

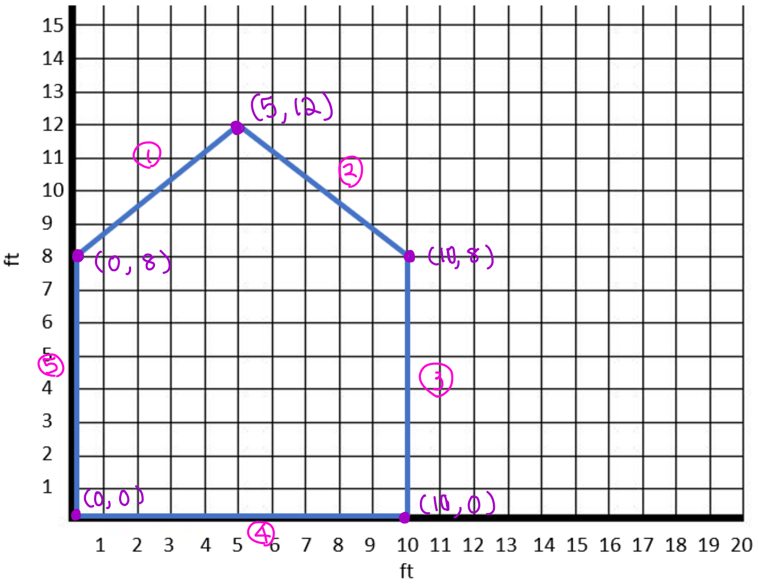
$A = b\ell = 10(8) = 80 \text{ units}^2$

Total: ① + ② $= 20 + 80$

$= 100 \text{ units}^2$

Key

NOTE: There are other ways to solve this problem.



Perimeter

① $\sqrt{(5-10)^2+(12-8)^2} = \sqrt{(-5)^2+(4)^2} = \sqrt{25+16} = \sqrt{41}$ units

② $\sqrt{(5-0)^2+(12-8)^2} = \sqrt{(5)^2+(4)^2} = \sqrt{25+16} = \sqrt{41}$ units

③ 8 units

④ 10 units

⑤ 8 units

Total: ① + ② + ③ + ④ + ⑤

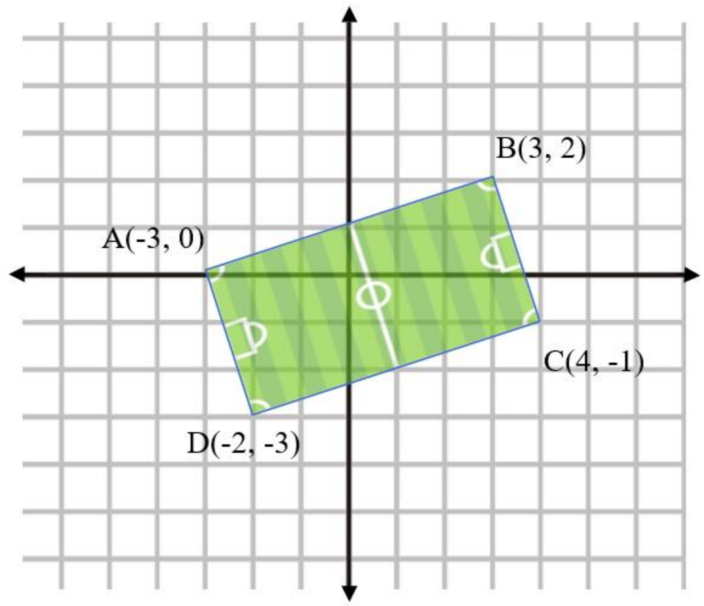
$= \sqrt{41} + \sqrt{41} + 8 + 10 + 8$

$= 2\sqrt{41} + 26$ units

Soccer Field Spruce Up

1. The school facilities manager wants to improve the soccer field. He has drawn a scaled down version of the field below.

a) First, he wants to plant new seed in the field. If one bag of grass seed covers 2 square units of the new scale, how many bags must he buy to cover the whole field? Explain.



b) Next, he wants to repaint the boundary lines. If he expects one can of paint to be enough for 6 units of the boundary line, how many cans should he buy?

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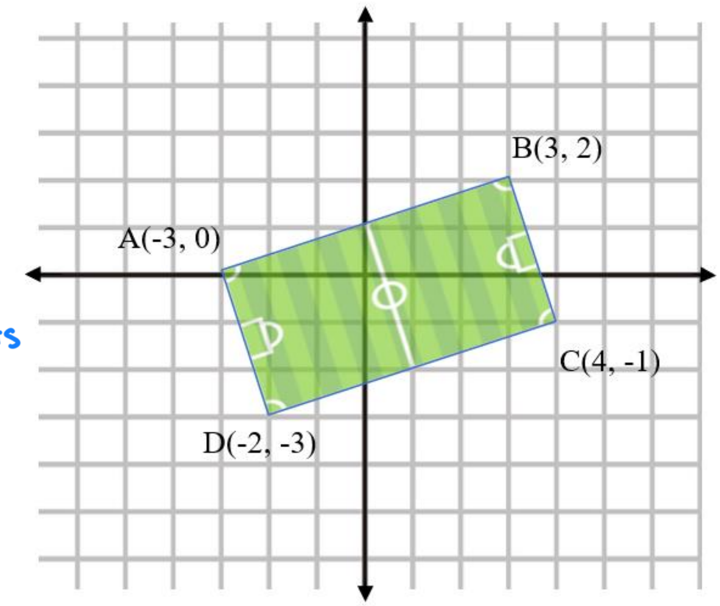


a) First, he wants to plant new seed in the field. If one bag of grass seed covers 2 square units of the new scale, how many bags must he buy to cover the whole field? Explain.

$AB = \sqrt{(-3-3)^2 + (0-2)^2}$
$= \sqrt{(-6)^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ units

base

$BC = \sqrt{(3-4)^2 + (2-(-1))^2}$
$= \sqrt{(-1)^2 + (3)^2} = \sqrt{1+9} = \sqrt{10}$

height

Area = base × height
$= 2\sqrt{10} \times \sqrt{10} = 20 \text{ units}^2$

$20 \div 2 = 10$

10 bags will be needed.

Explanations will vary.

b) Next, he wants to repaint the boundary lines. If he expects one can of paint to be enough for 6 units of the boundary line, how many cans should he buy?

$AB = 2\sqrt{10}$ units → length

$BC = \sqrt{10}$ units → width

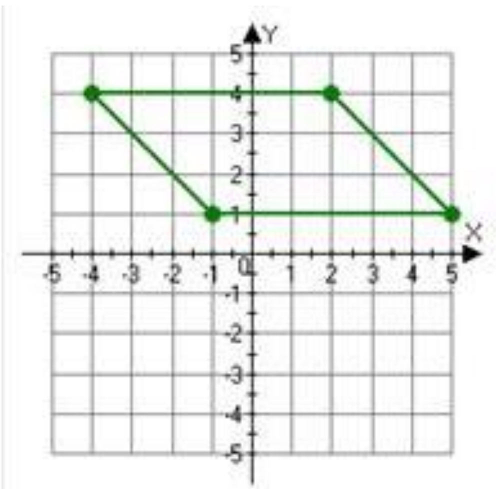
perimeter = 2 × length + 2 × width
$= 2(2\sqrt{10}) + 2(\sqrt{10})$
$= 4\sqrt{10} + 2\sqrt{10}$
$= 6\sqrt{10}$ units $\approx 18.97\ldots$

$6\sqrt{10} \div 6 \approx 3.1622$ He should buy 4 cans.

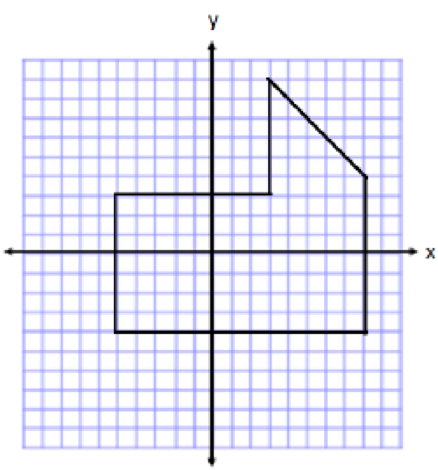
Area & Perimeter on the Coordinate Plane (HW)

Calculate the area and perimeter of the following objects:

1.



2.

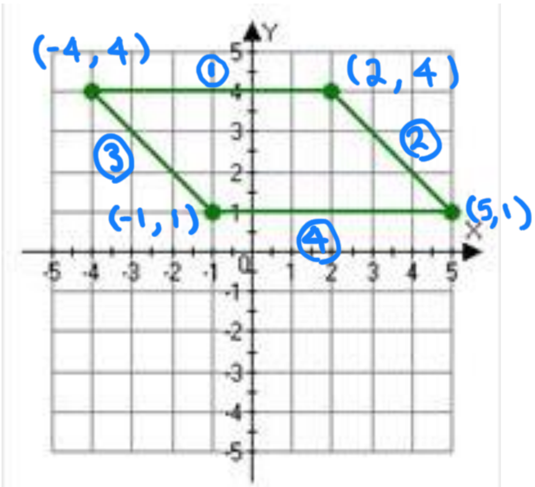


Area & Perimeter on the Coordinate Plane (HW)

Calculate the area and perimeter of the following objects:

Key (NOTE: There are other ways to solve these.)

1.



Perimeter

① 6 units

② $\sqrt{(2-5)^2+(4-1)^2}$

$=\sqrt{(-3)^2+(3)^2}$

$=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$ units

③ $\sqrt{(-4-(-1))^2+(4-1)^2}$

$=\sqrt{(-3)^2+(3)^2}=\sqrt{18}=3\sqrt{2}$ units

④ 6 units

Total: $6+3\sqrt{2}+3\sqrt{2}+6$

$=12+6\sqrt{2}$ units

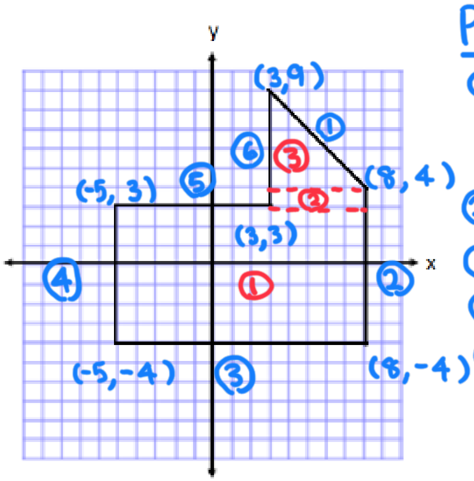
Area

height = 3 units

base = 6 units

$A=b\times h=3\times 6=18$ units^2

2.



Perimeter

① $\sqrt{(3-8)^2+(9-4)^2}$

$=\sqrt{(-5)^2+(5)^2}$

$=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$ units

② 8 units

③ 13 units

④ 7 units

⑤ 8 units

⑥ 6 units

Total: $5\sqrt{2}+8+13+7+8+6$

$=5\sqrt{2}+42$ units

Area

① height = 7 base = 13

$A=b\times h=7\times 13=91$ units^2

② height = 1 base = 5

$A=b\times h=1\times 5=5$ units^2

③ height = 5 base = 5

$A=\frac{1}{2}\times b\times h=\frac{1}{2}\times 5\times 5=\frac{25}{2}$ units^2

Total

$91+5+\frac{25}{2}$

$=108\frac{1}{2}$ units^2