Polygon Properties

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Target Grade: Geometry

Time Required: 75 minutes

Standards

Common Core Math Standards

- CCSS.MATH.CONTENT.HSG.MG.A.1
  Use geometric shapes, their measures, and their properties to describe objects.

Lesson Objectives

Students will:

- Describe the characteristics of a given polygon.
- Use the properties of the sum of the interior and exterior angles of a polygon to solve for missing values.

Central Focus

In this lesson, students recall their knowledge of polygons which will catch their attention because it allows them to draw from what they know, which is a different means of expression than is typically used. Polygons are all around students in the world (architecture, art, etc.). Understanding the characteristics of polygons will help students to describe the world around them.

Key Terms: convex, concave, polygon, interior angle, exterior angle, irregular polygon, regular polygon, diagonal, triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon, dodecagon

Background Information

This lesson begins with a warmup that asks students to brainstorm about what they already know about polygons. In previous grades, students will already have learned the names of polygons. They also already know that the sum of the angles of a triangle is 180°, which will be used as they calculate the sum of the interior angles of other polygons. This lesson builds on the students' prior knowledge of linear pairs, exterior angles, and symbols for marking congruent angles and line segments.

Prior to this lesson, teachers should be familiar with the following terms: convex, concave, polygon, interior angle, exterior angle, irregular polygon, regular polygon, diagonal, triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon, and dodecagon.

- Convex
  - Curved outwards. The green shape is convex but is not a polygon.
  - A polygon (which has straight sides) is convex when there are NO "dents" or indentations in it (no internal angle is greater than 180 degrees). The vertices of a convex polygon always point outwards. The pink shape is a convex polygon. (Convex Definition (Illustrated Mathematics Dictionary) (mathsisfun.com)_

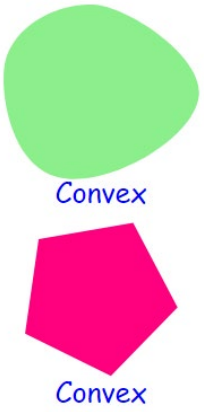


Figure 1: https://www.mathsisfun.com/definitions/convex.html

- Concave
  - Curved inwards. The green shape is concave but is not a polygon.
  - A polygon (which has straight sides) is concave when there are "dents" or indentations in it (where at least one internal angle is greater than 180 degrees). The vertices are inwards as well as outwards.
    (Concave Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

Figure 2: https://www.mathsisfun.com/definitions/concave.html

- Polygon
  - Polygons are 2-dimensional shapes.
  - They are made of straight lines, and the shape is "closed" (all the lines connect). (Polygons (mathsisfun.com))

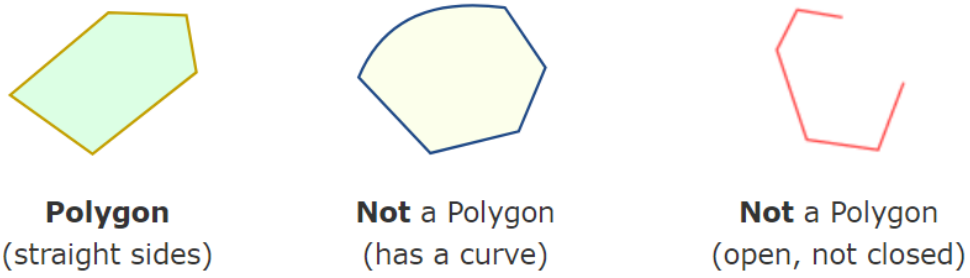


Figure 3: https://www.mathsisfun.com/geometry/polygons.html

- Interior Angle
  - An interior angle is an angle inside a shape. (Interior Angles of Polygons (mathsisfun.com))

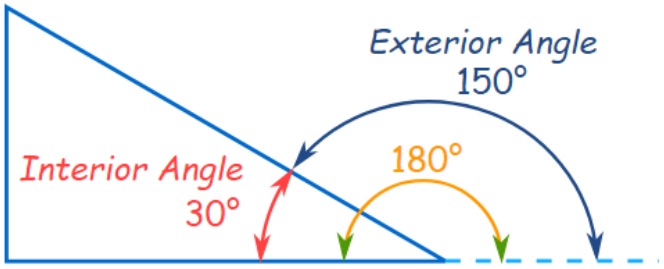


Figure 4: https://www.mathsisfun.com/geometry/interior-angles-polygons.html

- Exterior Angle
  - The exterior angle is the angle between any side of a shape, and a line extended from the next side. (Exterior Angles of Polygons (mathsisfun.com))

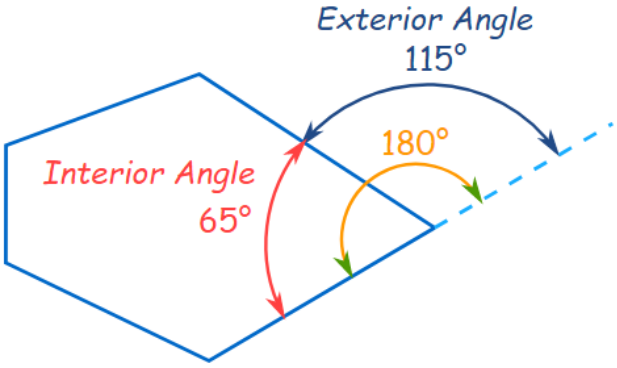


Figure 5: https://www.mathsisfun.com/geometry/exterior-angles-polygons.html

- Irregular Polygon
  - A polygon that does not have all sides equal and all angles equal. (Definition of Irregular Polygon (mathsisfun.com))

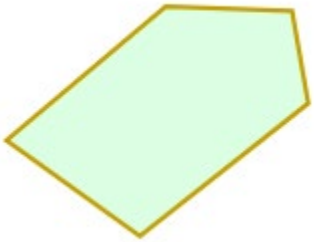


Figure 6: https://www.mathsisfun.com/definitions/irregular-polygon.html

- Regular Polygon
  - A polygon is regular when all angles are equal and all sides are equal. (Regular Polygon Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

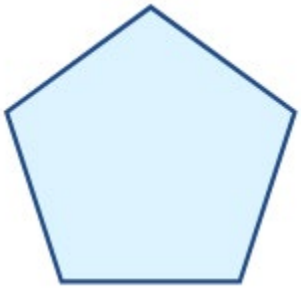


Figure 7: https://www.mathsisfun.com/definitions/regular-polygon.html

- Diagonal
  - A line segment that goes from one corner to another but is not an edge.
  - Directly joining any two opposite corners (called "vertices") which are not already joined by an edge, generates a diagonal. (Definition of Diagonal (mathsisfun.com))

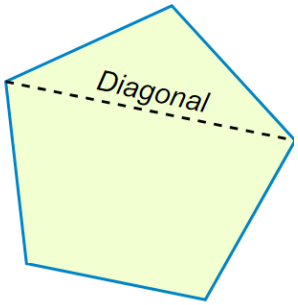


Figure 8: https://www.mathsisfun.com/definitions/diagonal.html:

- Triangle
  - A triangle has three sides and three angles.
  - The three angles always add up to 180 degrees.
  - Shape must be closed (Triangles - Equilateral, Isosceles and Scalene (mathsisfun.com))

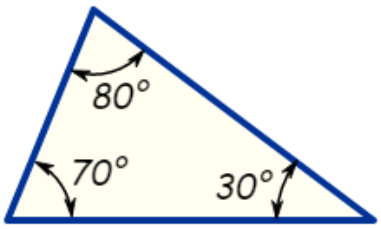


Figure 9: https://www.mathsisfun.com/triangle.html

- Quadrilateral
  - A quadrilateral has:
    - Four sides (edges)
    - Four vertices (corners)
    - Shape must be closed
    - Sum of interior angles of 360 degrees (Quadrilaterals - Square, Rectangle, Rhombus, Trapezoid, Parallelogram (mathsisfun.com))

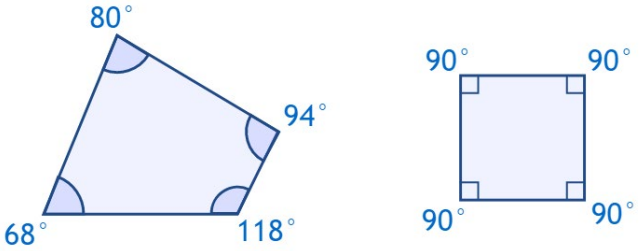


Figure 10: https://www.mathsisfun.com/quadrilaterals.html

- Pentagon
  - A pentagon has:
    - Sum of interior angles of 540 degrees
    - 5 straight sides
    - 5 diagonals
    - The shape also must be closed.

  (Pentagon (mathsisfun.com))

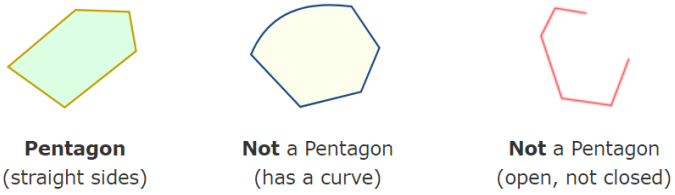


Figure 11: https://www.mathsisfun.com/geometry/pentagon.html

- Hexagon
  - A hexagon is a 6-sided polygon.
  - A hexagon has:
    - Sum of interior angles of 720 degrees
    - 9 diagonals

  (Hexagon (mathsisfun.com))

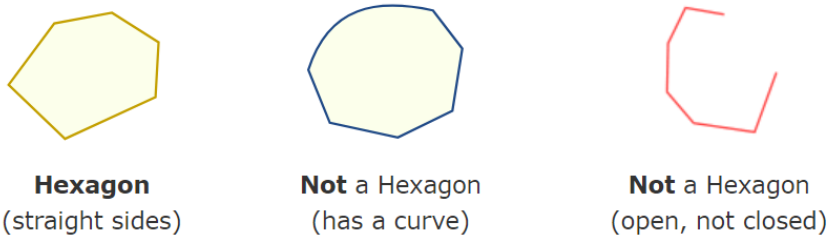


Figure 12: https://www.mathsisfun.com/geometry/hexagon.html

- Heptagon
  - A heptagon is a 7-sided polygon.
  - A heptagon has:
    - Sum of interior angles of 900 degrees
    - 14 diagonals

  (Heptagon Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

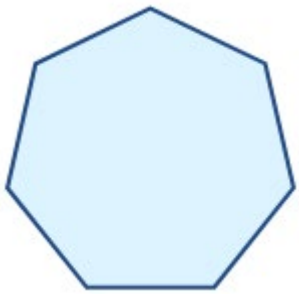


Figure 13: https://www.mathsisfun.com/definitions/heptagon.html

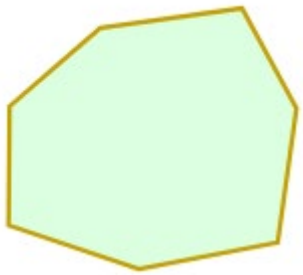


Figure 14: https://www.mathsisfun.com/definitions/heptagon.html

- Octagon
  - An octagon is an 8-sided polygon.
  - An octagon has:
    - Sum of interior angles of 1080 degrees
    - 20 diagonals

  (Octagon (mathsisfun.com))

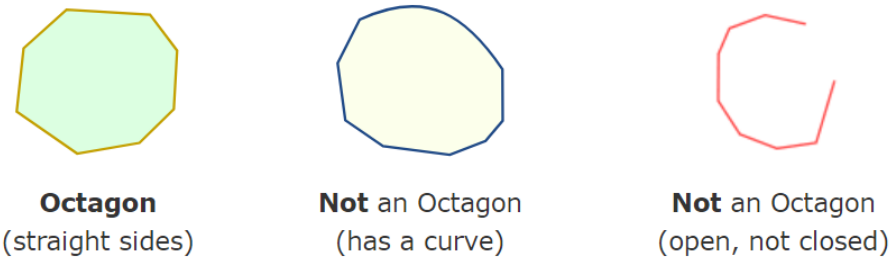


Figure 15: https://www.mathsisfun.com/geometry/octagon.html

- Nonagon
  - A 9-sided polygon.
    (Nonagon Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

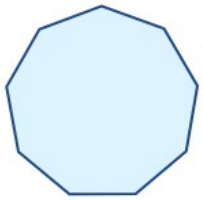


Figure 16: https://www.mathsisfun.com/definitions/nonagon.html

- Decagon
  - A 10-sided polygon.
    (Decagon Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

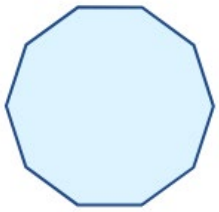


Figure 17: https://www.mathsisfun.com/definitions/decagon.html

- Dodecagon
  - A 12-sided polygon.
    (Definition of Dodecagon (mathsisfun.com))

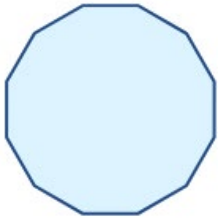


Figure 18: https://www.mathsisfun.com/definitions/dodecagon.html

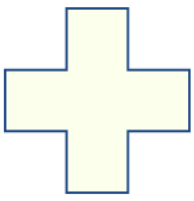


Figure 19: https://www.mathsisfun.com/definitions/dodecagon.html

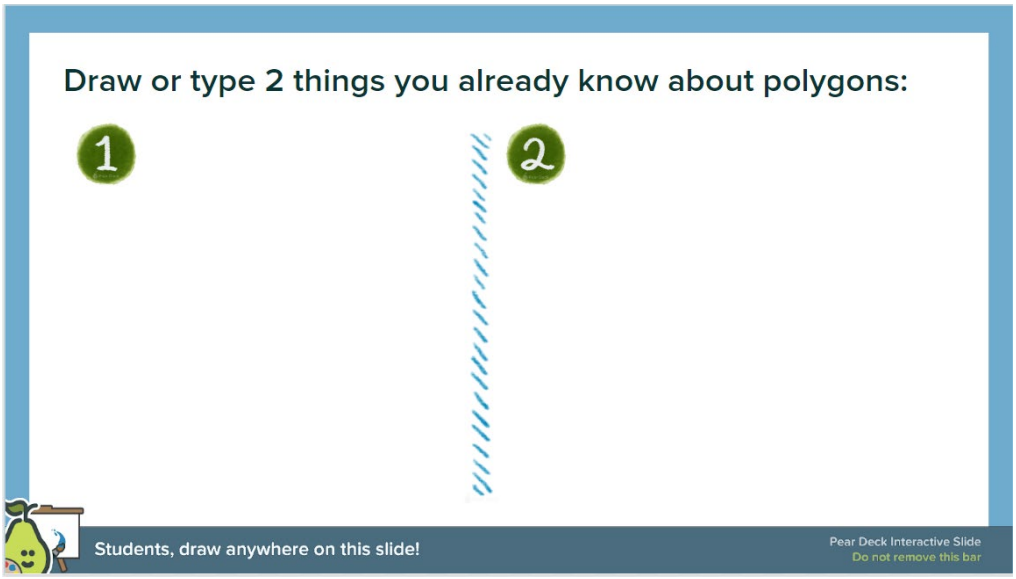
Materials

- Pear Deck: https://docs.google.com/presentation/d/1EHb6hOuFva3bPAiWKmCSpIPvaqqC0e5pYqesbaSRlH4/copy
- Polygon Properties Worksheet
- Polygon Properties Worksheet Key
- Polygon Sets (for individual students)
- Protractor
- Calculator (if needed)
- Exit Ticket
- Exit Ticket Key

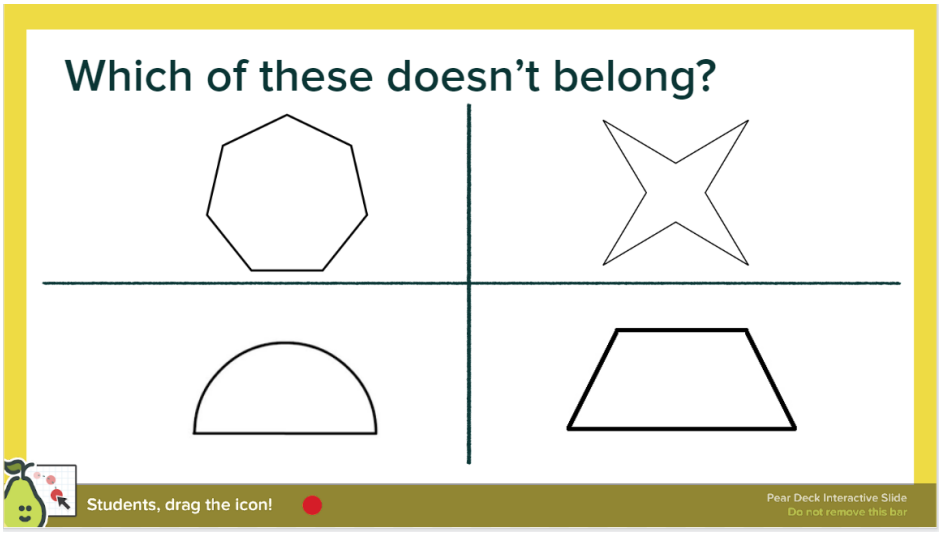
Instruction

Introduction (20 minutes)

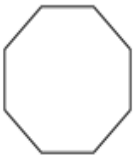
- Begin with a Pear Deck activity that activates the students' prior knowledge about polygons and the characteristics of polygons.
- Have students draw or type things they already know about polygons.



- Then, have them synthesize this information into a list of characteristics of polygons.
- Next, have the students use this list to identify shapes that are not polygons.



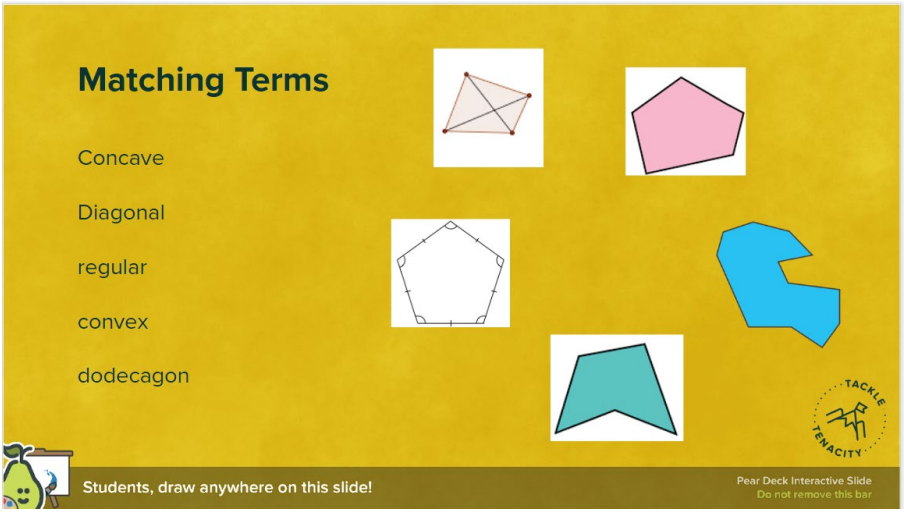
- Then, go over the new vocabulary of diagonal, regular, convex, and concave.
- Have the students use this vocabulary to solve 1-3 on the worksheet.
    - Is the shape a polygon? If yes, give its name, then state whether it is regular or irregular and whether it is convex or concave.

1. 

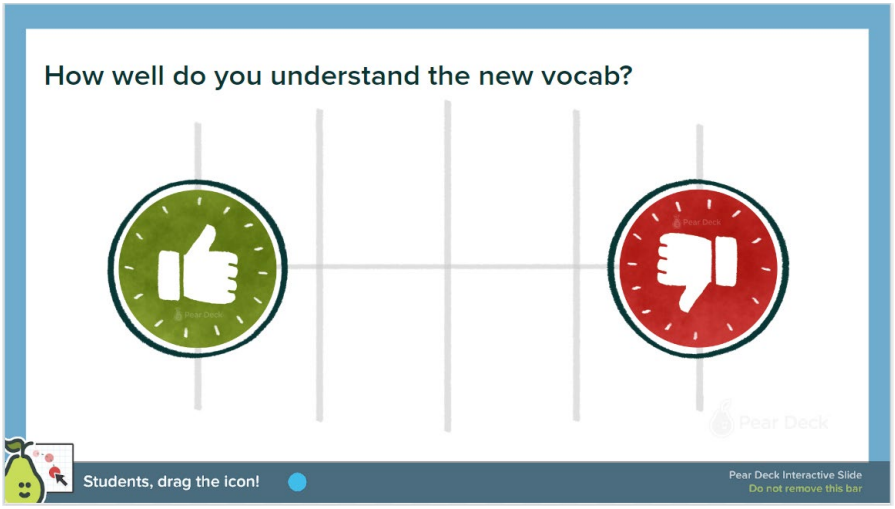
2. 

3. 

- Next, have students complete the matching activity on the Pear Deck.



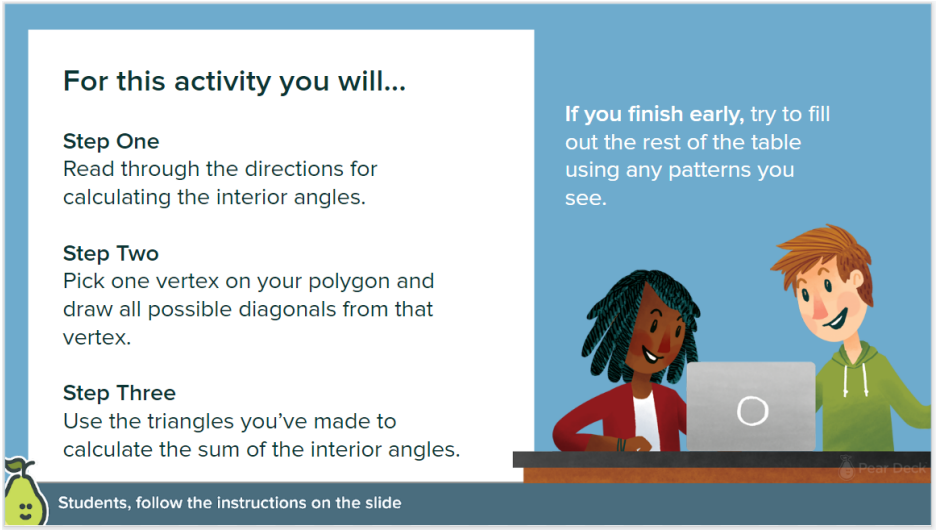
- Finally, instruct the students to reflect on how well they understand the new vocabulary.



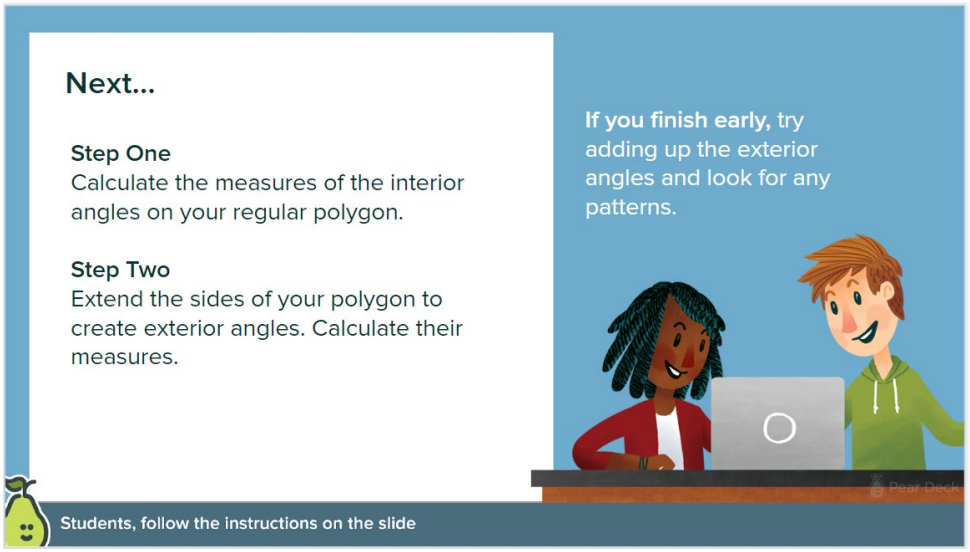
- Here are some questions to ask the students during the introduction:
  - Which diagonal in the concave polygon is outside the figure?
  - What does the term “adjacent” mean?
  - How do you know if a shape is a polygon?

Activity (45 minutes)

- Divide the class into groups of 3-4 students.
- Give each student a different set of polygons.
- Give one student a set of pentagons, another a set of hexagons, and so on.
  - Each set has one regular and one irregular polygon.
- Have the students follow the directions on the Interior Angles worksheet to fill out the table using the pattern they find for the first 5 shapes to find the values for the last 3.



- Finally, have the students generalize the pattern into an expression that can be used for any polygon.
- Then, instruct the students to apply this expression to find the interior angle sum of a 20-gon.
- Next, have the students use their tables to solve for the value of each interior angle of their regular polygon.
- Then, tell the students to calculate the measures of the exterior angles.



- Then, have the students use a protractor to measure the interior angles of their irregular polygon and then calculate the exterior angles.
- Have students then compare the sums of the exterior angles of each polygon in their group.
- Have a discussion which leads to the result that the sum of the exterior angles of a polygon, no matter the number of sides, is 360.
- Here are some questions to ask the students during the activity:
    - Why does it work to split the polygons into triangles and multiply the number of triangles by 180?
    - How can we calculate the measure of angles in a polygon if we know that the polygon is regular?
    - What is different about the sum of the interior angles and exterior angles of a polygon?

Closure (10 minutes)

- Hand out the exit ticket and have the students complete the exit ticket which requires them to apply their learning of the vocabulary and the sum of the interior and exterior angles.

Differentiation

Grouping

- Students will be in heterogeneous groups of 3-4 students.
- Each student will be responsible for calculating the sum of the interior and exterior angles of one type of polygon.

Students with learning disabilities

- Instead of having students use a protractor to measure the angles of the irregular polygon, the interior angle measures could be given to them. Then, they would only have to solve for the exterior angles using the fact that they are linear pairs.
- Provide the students with guided notes/worksheet.
- Write in angle measures on the irregular polygons instead of having students measure them, the teacher may give polygons with the diagonals already drawn on them.
- Allow students to work on a separate sheet of paper or work on a digital copy

Advanced Students

- Provide the student with a blank table on the Polygon Properties Worksheet, so that they can discover the pattern.
- Allow the students to work ahead.

Assessment

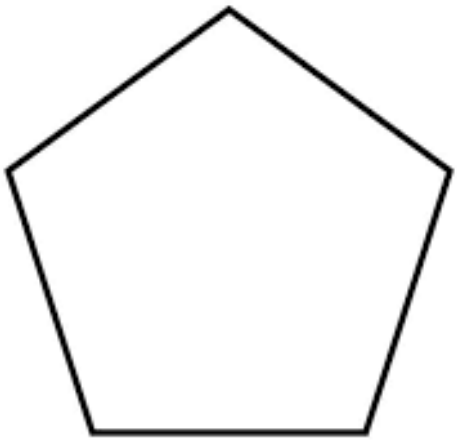
Formative Assessment

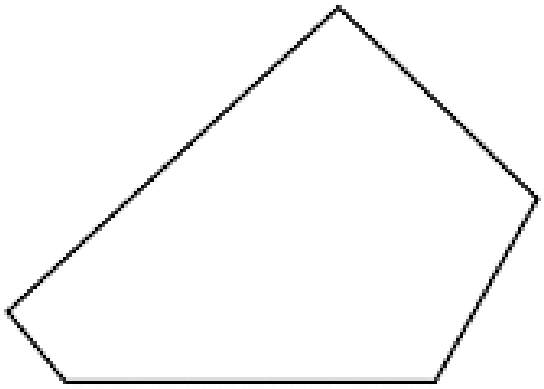
- The class discussion and the group discussions will allow the teacher to see what concepts the students understood as well as what they may be struggling with.

Summative Assessment

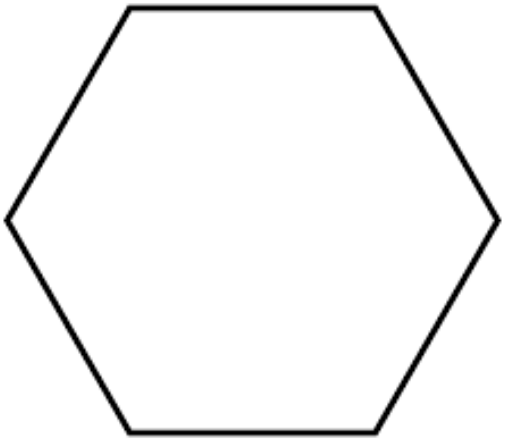
- This exit ticket assesses the students' ability to describe polygons accurately with academic language and to use interior and exterior angle theorems to solve for missing values.

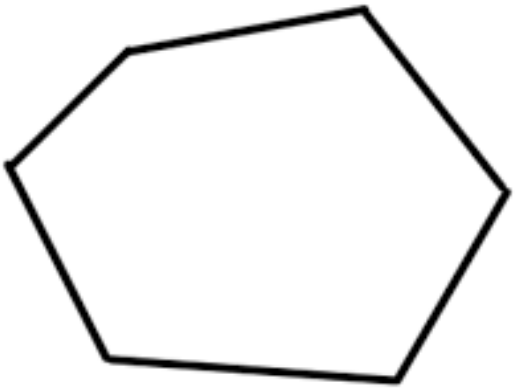
Pentagons:



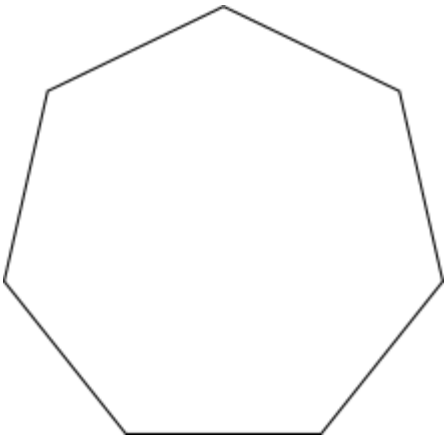


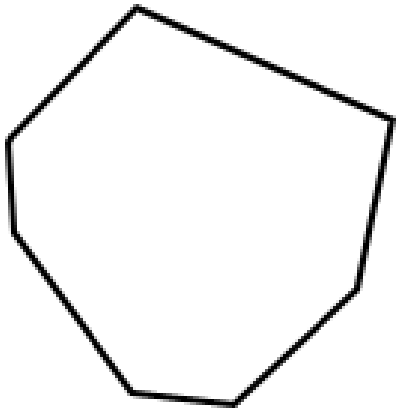
Hexagons:



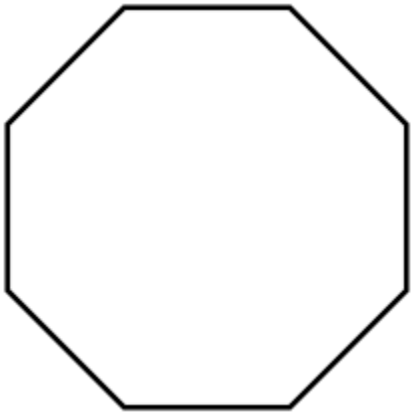


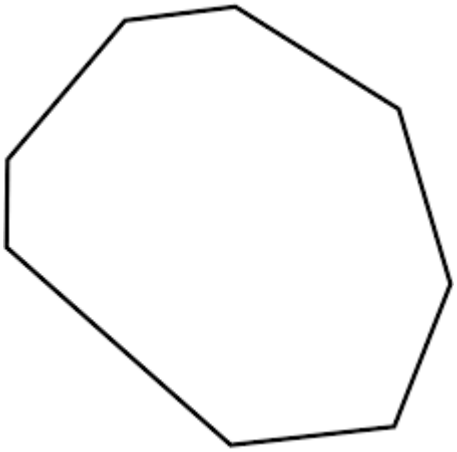
Heptagons:





Octagons:

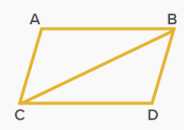
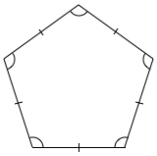
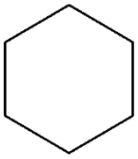
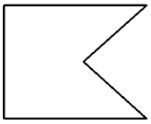




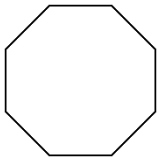
Polygon Properties

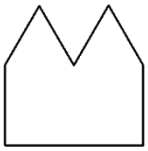
What is a polygon?

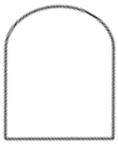
Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
n	n-gon

Diagonal: a line segment linking two non-adjacent vertices	Regular: all sides and angles are congruent	Convex: All the diagonals of the polygon lie in the interior of the polygon	Concave: all or part of at least one diagonal lies outside the polygon
			

Is the shape a polygon? If yes, give its name, then state whether it is regular or irregular and whether it is convex or concave.

1. 

2. 

3. 

Angle Sum Theorems

(Convex Polygons)

To find the sum of the interior angles of any polygon, do the following:

1. From one vertex, draw all possible diagonals.

2. Determine the number of triangles formed.

3. Multiply the number of triangles by 180.

Example:

Complete the table using this procedure

Polygon	Number of sides	Number of Triangles	Interior Angle Sum
Triangle	3	1	180˚
Quadrilateral	4		
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			
Dodecagon	12		
n-gon	n		

Try it out!

Use the information for the n-gon to determine the sum of the interior angles of a 20-gon.

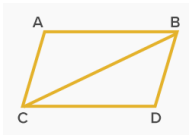
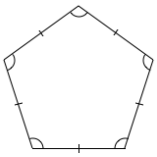
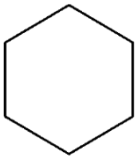
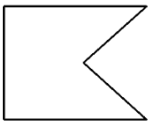
What is true about the sum of the exterior angles of a polygon?

Polygon Properties KEY

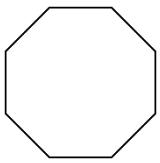
What is a polygon?

Answers may vary.

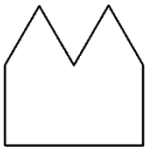
Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
n	n-gon

Diagonal: a line segment linking two non-adjacent vertices	Regular: all sides and angles are congruent	Convex: All the diagonals of the polygon lie in the interior of the polygon	Concave: all or part of at least one diagonal lies outside the polygon
			

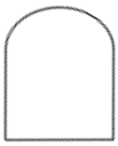
Is the shape a polygon? If yes, give its name, then state whether it is regular or irregular and whether it is convex or concave.

1. 

Yes; octagon, regular, convex

2. 

Yes; heptagon, irregular, concave

3. 

No

Angle Sum Theorems

(Convex Polygons)

To find the sum of the interior angles of any polygon, do the following:

1. From one vertex, draw all possible diagonals.

2. Determine the number of triangles formed.

3. Multiply the number of triangles by 180.

Example:

Complete the table using this procedure

Polygon	Number of sides	Number of Triangles	Interior Angle Sum
Triangle	3	1	180˚
Quadrilateral	4	2	360 degrees
Pentagon	5	3	540 degrees
Hexagon	6	4	720 degrees
Heptagon	7	5	900 degrees
Octagon	8	6	1080 degrees
Nonagon	9	7	1260 degrees
Decagon	10	8	1440 degrees
Dodecagon	12	10	1800 degrees
n-gon	n	n-2	(n-2) * 180 degrees

Try it out!

Use the information for the n-gon to determine the sum of the interior angles of a 20-gon.

(20 – 2) * 180 degrees = 3240 degrees

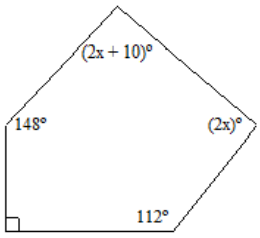
What is true about the sum of the exterior angles of a polygon?

It always adds up to 360 degrees

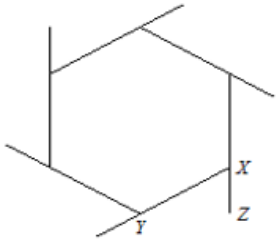
Exit Ticket

Name: _____

1. Find the value of x. (The diagram is not drawn to scale).



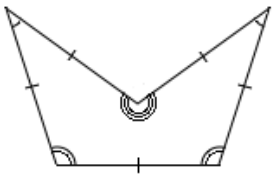
2. The door on a spacecraft is formed with 6 straight panels that overlap to form a regular hexagon. What is $m\angle YXZ$?



3. a) Name the polygon:

b) Circle the word that describes the polygon: convex or concave

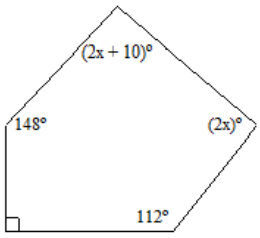
c) Circle the word that describes the polygon: regular or irregular



Exit Ticket

Name: _____KEY_____

1. Find the value of x. (The diagram is not drawn to scale).



180 degrees * 3 = 540 degrees

540 degrees – (90 degrees + 148 degrees + 112 degrees) = 190 degrees

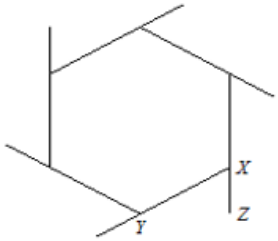
190 degrees = (2x + 10) degrees + 2x degrees

190 degrees = 4x + 10 degrees

180 degrees = 4x

x = 45 degrees

2. The door on a spacecraft is formed with 6 straight panels that overlap to form a regular hexagon. What is $m\angle YXZ$?



360 degrees / 6 = 60 degrees

3. a) Name the polygon:

b) Circle the word that describes the polygon: convex or concave

c) Circle the word that describes the polygon: regular or irregular