Carbon Dating and Logarithmic Functions: Understanding Exponential Decay in Nuclear Science

Submitted by: Emily McDonald, Science and Math
University High School, Chattanooga, TN

Target Grade: 10th–11th Grade Chemistry, Physics, Biology, Algebra 2, or Pre-Calculus

Time Required: 90 minutes

Standards

Common Core State Standards (CCSS):

- HSF.LE.A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- HSF.BF.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Next Generation Science Standards (NGSS):

- HS-PS1-8: Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the processes of fission, fusion, and radioactive decay.

Lesson Objectives

Students will:

- Understand the principles of radioactive decay and its application in carbon dating.
- Apply logarithmic functions to solve problems involving carbon dating.
- Interpret the function of carbon-14's half-life in calculating the age of fossils and artifacts.

Central Focus

In this cross-curricular lesson, students will investigate the connection between nuclear science, mathematics, and archaeology through the study of carbon-14 dating. Students will learn how logarithmic functions model radioactive decay and how these models are used to determine the age of ancient artifacts. Through hands-on practice, simulations, and real-world data, students will develop both mathematical and scientific literacy.

Key Terms: Radiometric dating, radioactive decay, half-life, exponential decay, archaeological dating, nuclear decay, nuclear science

Background Information

Teacher Background Information

Exponential and Logarithmic Functions: Be comfortable explaining how exponential decay works and how logarithms serve as the inverse of exponential functions. You should be able to guide students through converting between exponential and logarithmic forms and solving for time using the carbon dating formula.

Half-Life and Radioactive Decay: Understand the concept of half-life as the time it takes for half of a radioactive substance to decay. Be familiar with how this predictable decay pattern is modeled mathematically and how it applies to real-world scenarios like radiometric dating.

Carbon-14 and Archaeological Dating: Know how carbon-14 is formed, how it decays, and why it is useful for dating organic materials. You should be able to explain why carbon-14 is effective for dating artifacts up to about 50,000 years old and how its half-life (approximately 5,730 years) factors into age calculations.

PhET Simulation and Desmos Graphing Calculator: Be prepared to demonstrate or support students in using the PhET Radioactive Dating Game to explore decay processes interactively. Also, be familiar with Desmos or other graphing tools to help students visualize exponential decay and interpret data.

Student Background Information

Exponents and Exponential Functions: Students should understand how exponential growth and decay work, including how to evaluate expressions.

Basic Logarithmic Concepts: Students should be introduced to logarithms as the inverse of exponentials and be able to interpret and evaluate simple logarithmic expressions, even if they are still developing fluency.

Atomic Structure: Students should know the basic components of an atom, protons, neutrons, and electrons, and understand how isotopes differ in neutron count.

Isotopes: Students should understand that isotopes are atoms of the same element with different numbers of neutrons, and that some isotopes (like carbon-14) are unstable and undergo radioactive decay over time.

Materials

- Carbon Dating Calculation Practice Handout
- Internet-capable devices
- Online simulation: PhET Radioactive Dating Game
- Video: *How Does Radiocarbon Dating Work?* – Scientific American
- Desmos online graphing calculator (optional)

Instruction

Opening (15 minutes)

- Pose the guiding question: "How do scientists know if an artifact is thousands of years old?"
- Facilitate a Think-Pair-Share by asking students to brainstorm possible dating methods and discuss with a partner.
  - During the class share-out, use student responses to introduce the concept of carbon dating, emphasizing its connection to both archaeology and mathematics.
- Show the video *How Does Radiocarbon Dating Work?* to introduce the carbon dating.
  - Carbon dating is a nuclear science application that uses the predictable decay of carbon-14 to estimate the age of organic materials.
- After the video, facilitate a brief discussion to address any questions students may have about the video.

Activity (50–60 minutes)

Math Foundation (10–15 minutes)

- Present the carbon-14 decay formula.
  - $N(t) = N_0 * e^{-kt}$
    - $N(t)$: The amount of carbon-14 remaining at time t
    - N_0: The original amount of carbon-14 (at time $t = 0$)
    - *k*: The decay constant (specific to carbon-14, approximately 0.000121)
    - *t*: Time elapsed (usually in years)
    - *e*: Euler's number (approximately 2.718), the base of the natural logarithm
- Demonstrate or guide students in rearranging the formula into logarithmic form to solve for time *t.*
  - Step 1: Starting with $N(t) = N_0 * e^{-kt}$, divide both sides by N_0.
    - $\frac{N(t)}{N_0} = e^{-kt}$
  - Step 2: Take the natural logarithm (ln) of both sides
    - We take the natural logarithm (ln) when solving the carbon dating formula because the equation involves an exponential function with base *e,* and the inverse of an exponential function is a logarithmic function.
    - $\ln\left(\frac{N(t)}{N_0}\right) = \ln\left(e^{-kt}\right)$
  - Step 3: Apply the logarithmic identity $\ln(e^x) = x$.
    - $\ln\left(\frac{N(t)}{N_0}\right) = -\mathrm{kt}$
  - Step 4: Solve for *t.*
    - $t = \frac{-1}{k} * \ln\left(\frac{N_t}{N_{(0)}}\right)$

- Step 5: Use the logarithmic identity $\ln\left(\frac{a}{b}\right) = -\ln\left(\frac{b}{a}\right)$.
  - $t = \frac{1}{k} * \ln\left(\frac{N_0}{N_{(t)}}\right)$
- Explain how logarithms are used to "undo" exponential decay, reinforcing their role as the inverse of exponential functions.

Guided Example (10 minutes)

- As a class, work through a sample problem by guiding students through each step of the calculation. Problem: If an artifact has 25% of its original carbon-14, how old is it?
  - Step 1: Understand the formula.
    - $t = \frac{1}{k} * \ln\left(\frac{N_0}{N_{(t)}}\right)$
      - $N(t)$: The amount of carbon-14 remaining at time t
      - N_0: The original amount of carbon-14 (at time $t = 0$)
      - k: The decay constant (specific to carbon-14, approximately 0.000121)
      - t: Time elapsed (usually in years)
      - e: Euler's number (approximately 2.718), the base of the natural logarithm
  - Step 2: Plug in variables/what you know
    - We're told that 25% of the original carbon-14 remains. That means $N(t) = 0.25 * N_0$ so the ratio $\frac{N_0}{N(t)} = \frac{N_0}{0.25 * N_0} = 4$.
  - Step 3: Plug into the formula: $t = \frac{1}{0.000121} * \ln(4) = 11{,}454\ years$

Partner Practice (15–20 minutes)

- Distribute the Carbon Dating Calculation Practice Handout and instruct students to work in pairs to solve problems using real or simulated archaeological data.
  - Provide support with graphing tools like Desmos if needed to visualize exponential decay.

Digital Simulation (15 minutes)

- As students finish their packet, direct them to the [PhET Radioactive Dating Game].
- Instruct students to explore the concepts of half-life, decay, and radioactivity through interactive simulations.
- Encourage them to observe how decay occurs at the particle level and how it relates to larger samples.

Class Discussion (5–10 minutes)

- Facilitate discussion using the following questions to identify misconceptions and review key concepts:
  - How does carbon-14 decay relate to logarithmic functions?

- How does this mathematical model help archaeologists understand history?

Closure – Exit Ticket (5 minutes)

- Ask students to solve: If a bone fragment is found with 12.5% of its original carbon-14 remaining, calculate its approximate age.
  - $t = \frac{1}{k} * \ln\left(\frac{N_0}{N_{(t)}}\right)$
    - $N(t)$: The amount of carbon-14 remaining at time t
      - In this problem $N(t) = 0.125 * N_0$ $(12.5\%\ remaing)\ so\ \frac{N_0}{N(t)} = \frac{1}{0.125} = 8$
    - N_0: The original amount of carbon-14 (at time $t = 0$)
    - k: The decay constant (specific to carbon-14, approximately 0.000121)
    - t: Time elapsed (usually in years)
    - e: Euler's number (approximately 2.718), the base of the natural logarithm
  - $t = \frac{1}{0.000121} * \ln(8) = 17{,}182\ years$
- Collect responses to assess individual understanding.

Differentiation

- Advanced Learners:
  - Assign research on other isotopic dating methods (e.g., uranium-lead, potassium-argon).
  - Provide multi-step problems involving complex decay scenarios.
- Struggling Learners:
  - Offer step-by-step guides for formula use.
  - Use simplified data sets and gradually increase complexity.
  - Provide pre-formatted spreadsheets for graphing and calculations.
- English Language Learners:
  - Provide illustrated vocabulary cards (e.g., half-life, decay, isotope).
  - Pair with bilingual peers or provide translated materials as needed.
- Extensions and Modifications:
  - Allow oral presentations or digital portfolios as alternative assessments.
  - Offer additional real-world scenarios for students to analyze.

Assessment

Formative Assessments:

- Observe students during guided and partner practice to assess understanding.
- Review exit ticket responses for accuracy and application of the decay formula.

Summative Assessments (completed during next class):

- Calculation Quiz (Math): Include logarithmic problems based on carbon dating.

- Simulation Lab Report (Science): Have students write a brief report interpreting simulation results.
- Project: Students create an “archaeologist’s field notebook” with dating scenarios and original problems.

Lesson Handout Name: ___________________________

Carbon Dating Calculation Practice Handout

Instructions

You will use the formula for carbon-14 dating to calculate the approximate ages of artifacts, graph the decay of carbon-14 over time, and interpret decay curves. Remember that the half-life of carbon-14 is 5,730 years.

Why 5,730 years? Carbon-14 is considered to have a half-life of 5,730 years because this is the amount of time it takes for half of the carbon-14 atoms in a sample to decay into nitrogen-14 atoms, which is a stable isotope. Basically, after 5,730 years, only half the original amount of carbon-14 remains in a sample, making it a key factor in radiocarbon dating.

The 5,730-year half-life of carbon-14 plays a crucial role in the decay formula because it determines the decay constant *k*, which is the rate at which carbon-14 decays over time.

$$k = \frac{\ln(2)}{half - life}$$

For carbon 14, $k = \frac{\ln(2)}{5{,}730\ years} = 0.000121$

We use ln(2) in the decay constant formula because half-life is the time it takes for a substance to decay to half of its original amount, and ln(2) is the natural logarithm of 2, which mathematically represents that "halving" process.

Carbon-14 Decay Formula:

$$N(t) = N_0 * e^{-kt}$$

$N(t)$: The amount of carbon-14 remaining at time t
N_0: The original amount of carbon-14 (at time $t = 0$)
k: The decay constant (specific to carbon-14, approximately 0.000121)
t: Time elapsed (usually in years)
e: Euler's number (approximately 2.718), the base of the natural logarithm

Part 1: Calculating Ages from Remaining Carbon-14

Problem 1: An artifact has 60% of its original carbon-14 remaining. Use the decay formula to calculate the approximate age of the artifact.

Problem 2: A bone fragment is found with 10% of its original carbon-14 remaining. How old is the fragment? Show all your work.

Problem 3: A wooden tool contains 25% of its original carbon-14. Using the carbon-14 decay formula, determine the approximate age of the tool. Show your steps.

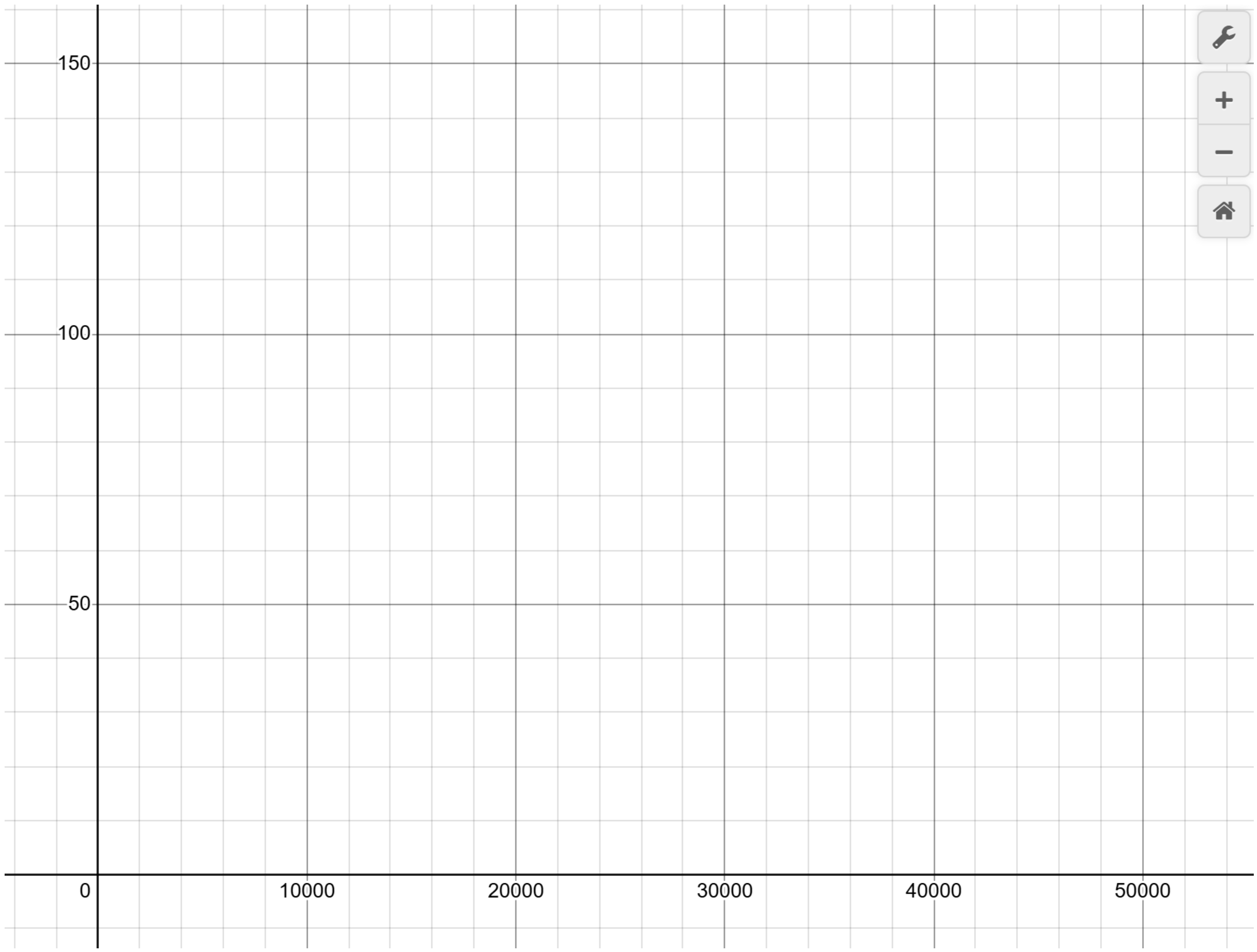
Problem 4: A piece of fabric has only 5% of its original carbon-14 remaining. Calculate the approximate age of the fabric.

Part 2: Graphing Carbon-14 Decay

1. *Table of Decay Values:* Complete the following table for the percentage of carbon-14 remaining over time, given that the half-life is 5,730 years. This table will help you plot the decay curve.

Time (years)	***Percentage of C-14 Remaining (%)***
0	
5,730	
11,460	
17,190	
22,920	
28,650	
34,380	

2. *Plotting the Decay Curve:* Using the completed table above, plot each point on the graph below. Label the *x*-axis as "Time (years)" and the *y*-axis as "Percentage of C-14 Remaining (%)". Draw a smooth curve that connects the points. This curve should represent the exponential decay of carbon-14 over time.



Part 3: Interpreting the Decay Curve
Answer the following questions based on your decay curve.

Decay Interpretation Questions

a) After 5,730 years, what percentage of the original carbon-14 remains?

b) How much carbon-14 remains after 22,920 years?

c) Based on your graph, estimate how long it would take for only 1% of the original carbon-14 to remain.

Real-World Connections

a) Archaeologists discover a fossil with about 12.5% of its original carbon-14 remaining. Based on your graph or calculations, estimate the age of the fossil.

b) Explain why the carbon-14 dating method becomes less accurate for materials older than 50,000 years.

Reflection Question

Describe how this graph helps you understand the concept of exponential decay. Use terms like half-life, exponential, and logarithmic function.

Lesson Handout - Sample Student Work/Key Name: ___________________________

Carbon Dating Calculation Practice Handout

Instructions

You will use the formula for carbon-14 dating to calculate the approximate ages of artifacts, graph the decay of carbon-14 over time, and interpret decay curves. Remember that the half-life of carbon-14 is 5,730 years.

Why 5,730 years? Carbon-14 is considered to have a half-life of 5,730 years because this is the amount of time it takes for half of the carbon-14 atoms in a sample to decay into nitrogen-14 atoms, which is a stable isotope. Basically, after 5,730 years, only half the original amount of carbon-14 remains in a sample, making it a key factor in radiocarbon dating.

Carbon-14 Decay Formula:

Carbon-14 Decay Formula:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

where:

- $A =$ remaining amount of carbon-14
- $A_0 =$ initial amount of carbon-14
- $t =$ time elapsed (in years)
- $5730 =$ half-life of carbon-14

For calculating t when given A and A_0, you can rearrange this as: $t = -5730 \times \log_2\left(\frac{A}{A_0}\right)$

Part 1: Calculating Ages from Remaining Carbon-14

Problem 1: An artifact has 60% of its original carbon-14 remaining. Use the decay formula to calculate the approximate age of the artifact.

Given: $\frac{A}{A_0} = 0.60$

Formula:

$t = -5730 \cdot \log_2(0.60)$

Calculation:

$t = -5730 \cdot \frac{\ln(0.60)}{\ln(0.5)}$

$t \approx 3,726$ years

Answer: 3,726 years

Problem 2: A bone fragment is found with 10% of its original carbon-14 remaining. How old is the fragment? Show all your work.

Given: $\frac{A}{A_0} = 0.10$

Formula:

$t = -5730 \cdot \log_2(0.10)$

Calculation:

$t \approx 18,985$ years

Answer: $\approx 18,985$ years

Problem 3: A wooden tool contains 25% of its original carbon-14. Using the carbon-14 decay formula, determine the approximate age of the tool. Show your steps.

- Given: $\frac{A}{A_0} = 0.25$
- Formula:

$t = -5730 \cdot \log_2(0.25)$

- Calculation:

$t \approx 11,460$ years

Answer: 11,460 years

Problem 4: A piece of fabric has only 5% of its original carbon-14 remaining. Calculate the approximate age of the fabric.

Given: $\frac{A}{A_0} = 0.05$

Formula:

$t = -5730 \cdot \log_2(0.05)$

Calculation:

$t \approx 25,315$ years

Answer: 25,315 years

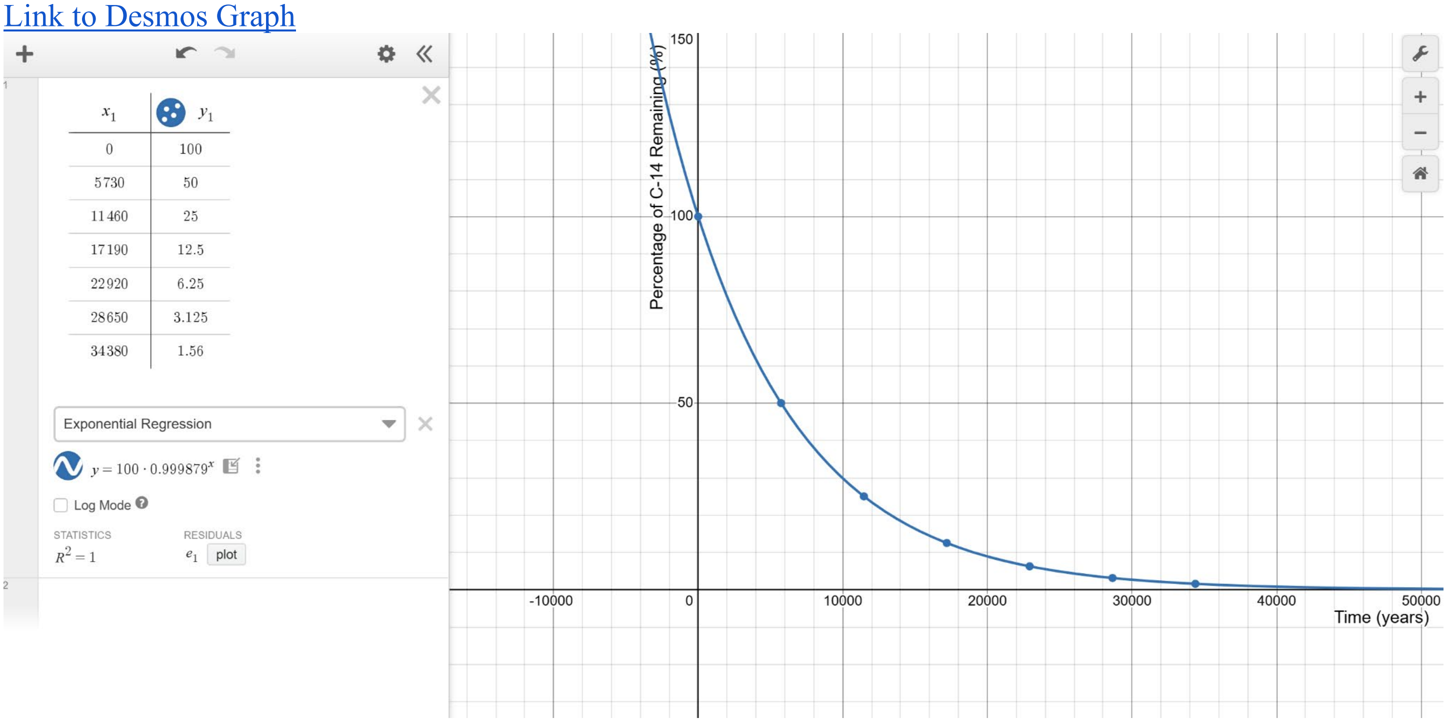
Part 2: Graphing Carbon-14 Decay

1. *Table of Decay Values:* Complete the following table for the percentage of carbon-14 remaining over time, given that the half-life is 5,730 years. This table will help you plot the decay curve.

Time (years)	***Percentage of C-14 Remaining (%)***
0	100
5,730	50
11,460	25
17,190	12.5
22,920	6.25
28,650	3.125
34,380	1.56

2. *Plotting the Decay Curve:* Using the completed table above, plot each point on the graph below. Label the x-axis as "Time (years)" and the y-axis as "Percentage of C-14 Remaining (%)". Draw a smooth curve that connects the points. This curve should represent the exponential decay of carbon-14 over time.

Link to Desmos Graph



Part 3: Interpreting the Decay Curve

Answer the following questions based on your decay curve.

Decay Interpretation Questions

a) After 5,730 years, what percentage of the original carbon-14 remains?
Answer: 50%

b) How much carbon-14 remains after 22,920 years?
Answer: 6.25%

c) Based on your graph, estimate how long it would take for only 1% of the original carbon-14 to remain.
Answer: Approximately 34,380 years

Real-World Connections

a) Archaeologists discover a fossil with about 12.5% of its original carbon-14 remaining. Based on your graph or calculations, estimate the age of the fossil.
Answer: The fossil would be approximately 17,190 years old.

b) Explain why the carbon-14 dating method becomes less accurate for materials older than 50,000 years.
Sample Response: Carbon-14 dating becomes less accurate for materials older than 50,000 years because the remaining carbon-14 levels become too low to measure accurately, leading to greater uncertainty in age estimations.

Reflection Question

Describe how this graph helps you understand the concept of exponential decay. Use terms like half-life, exponential, and logarithmic function.
Sample Response: The graph shows that as time increases, the percentage of carbon-14 decreases exponentially, meaning it declines at a rate proportional to its current amount. Each half-life represents a period in which half of the remaining carbon-14 decays, creating a smooth curve. By using logarithmic functions, we can solve for time when given a certain percentage, as logarithms are the inverse of exponential functions. This visualization helps illustrate how carbon-14 decays predictably over time and enables archaeologists to date ancient artifacts.