How Tall is that Tree? We need Trigonometry!

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Target Grade: 9th-10th grade; Geometry

Time Required: 60 minutes

Standards

Math Common Core State Standard

- CCSS.Math.Content.HSG-MG.A.1: Use geometric shapes, their measures, and their properties to describe objects.
- CCSS.Math.Content.HSG-SRT.C.8: Define trigonometric ratios and solve problems involving right triangles. Use trigonometric ratios and Pythagorean Theorem to solve right triangles.

Lesson Objectives

Students will:

- Interpret and draw triangles to represent measurement scenarios.
- Use the Pythagorean theorem to calculate the height of objects.
- Use trigonometric ratios to calculate the height of objects.
- Understand the relationship between angles, distances, and height.

Central Focus

How tall is that tree? How high is the flag? In this hands-on-activity, students will take their learning about trigonometric ratios and apply it to the real world! Students will create a simple surveying tool called a clinometer, which will allow them to measure the angle to the top of a real-world object, such as their house, a telephone pole, or a tree. The students will then use this measurement, the horizontal distance to the base of the object, and their knowledge of basic trigonometry to calculate the height of these objects.

Key terms: trigonometry applications, Pythagorean theorem, height calculation, real-world problems, measuring angles, triangle measurement, trigonometric equations.

Background Information

Student Background Information

Prior to this lesson, students must be familiar with how to use tangent to be able to solve for a missing side. Students must also be familiar with the Pythagorean Theorem ($a^2 + b^2 = c^2$), which will be used to solve for the distance to the top of the object (the hypotenuse).

- Tangent Ratio
  - $\tan(angle) = \frac{opposite\ side}{adjacent\ side}$

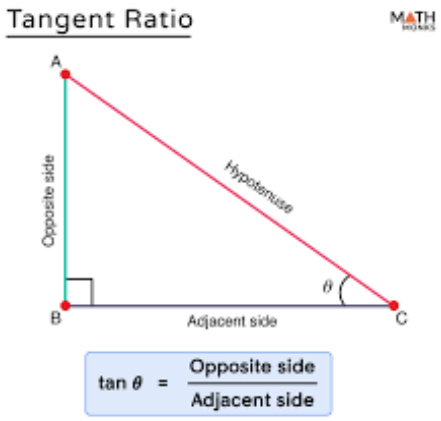


Figure 1: St. Louis Priory School

- Pythagorean Theorem
  - $a^2 + b^2 = c^2$
  - It is used to find the missing side of a right triangle.

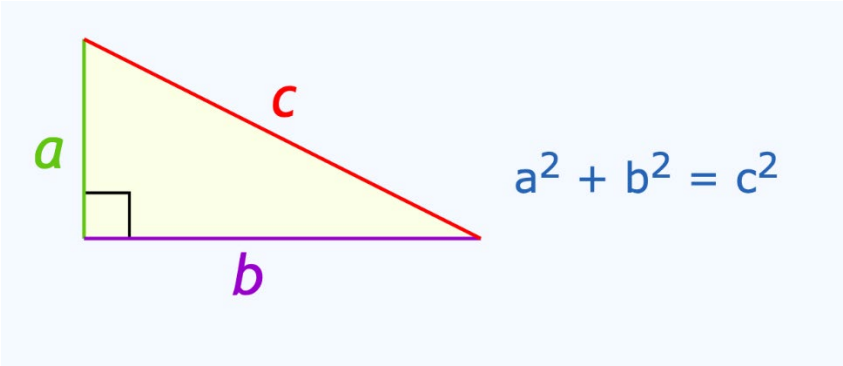


Figure 2: https://www.mathsisfun.com/pythagoras.html

Teacher Background Information

Prior to this lesson, teachers should be familiar with the term “angle of elevation.” Teachers need to know what a clinometer is and its purpose. Students will measure the angle of elevation (using the clinometer) and will measure the length of the “adjacent side” (the horizontal distance to the object) in order to solve for the length of the “opposite side”. Thus, teachers will need to be familiar with manipulating the equation such as

$$\tan(angle)\, x\, adjacent\ side = opposite\ side$$

- Angle of Elevation
  - This is defined as an angle formed between the line of sight and the horizontal line (from the eye).
  - Here the eye is acting as the vertex of the angle.

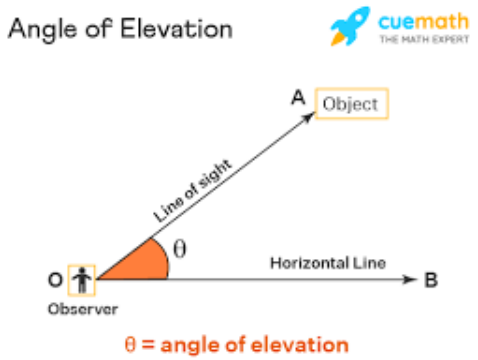


Figure 3: https://www.cuemath.com/trigonometry/angle-of-elevation/

- Clinometer
  - A simple device that can be used to measure the altitude of an object. (https://astronavigationdemystified.com/making-a-mini-clinometer/)

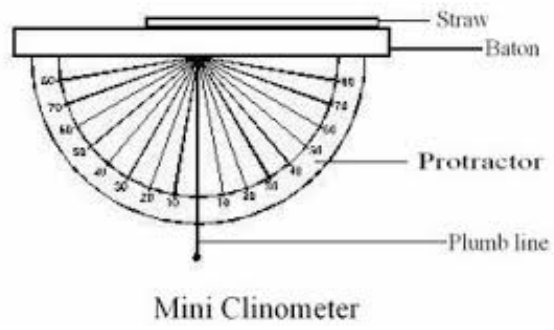


Figure 4: https://astronavigationdemystified.com/making-a-mini-clinometer/

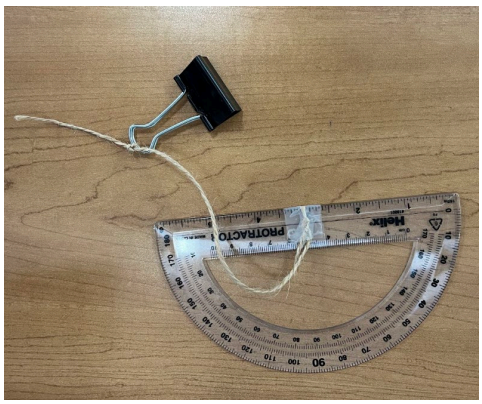
Materials

- Protractor
- Tape
- Straw
- String
- Weight (examples: binder clip or washer)
- Cardboard
- Tape measure
- Scientific or graphing calculator
- Trigonome-Tree Worksheet

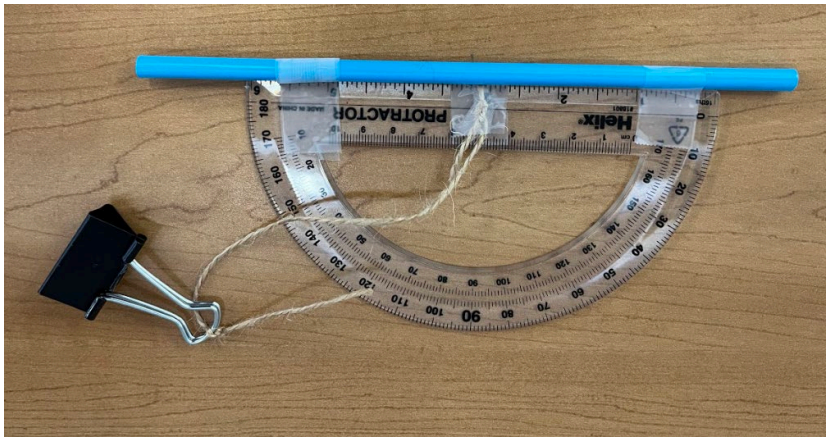
Instructions

Introduction (10 minutes)

- Pose the question: "How can someone determine the height of an object if it goes beyond their ability to measure it with a ruler or yard stick?"
- Facilitate a discussion on different ways to take measurements of large objects/structures. Students may come up with creative ways or name different tools that could be used.
- Show and introduce the purpose of a clinometer. Share that a clinometer is a device used for measuring angles of slope, elevation, or depression of an object with respect to gravity. It is commonly used in the surveying and construction fields.
- Pass out string, straw, protractor, tape, and a binder clip (weight)
- Have the students create the clinometer. Share the following steps.
    - Step 1: Cut the string to approximately 12 inches long.
    - Step 2: Attach the string to the center of the protractor. If the protractor has a hole, you may put the string through the hole and tie it. Secure the string with tape.
    - Step 3: Attach the weight to the other end of the string.



- Step 4: Attach the straw along the bottom edge of the protractor. Secure the straw with tape.



Activity (40 minutes)

- Pass out the Trigonome-Tree Worksheet.

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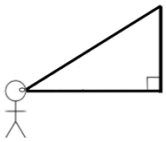
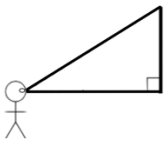
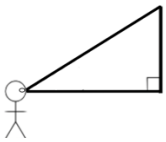
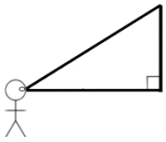
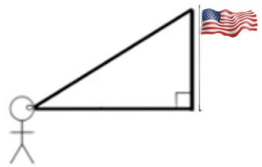
Trigonome-Tree Worksheet

Object	Triangle representation	Horizontal Distance	Angle of elevation	Equation used to solve	Height of the object	Distance to the top of the tree
						
						
						
						

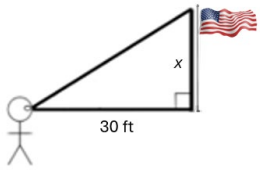
- Have the students calculate the height of real-life objects.
    - Step 1: Have the students choose a tall object to measure. Perhaps start with a tree!
    - Step 2: Choose a spot to stand where you can see the top of the object. The horizontal distance to the object should be relatively flat and unobstructed.
    - Step 3: Draw the triangle that represents the scenario on the chart on the Trigonome-Tree Worksheet.
    - Step 4: Measure the horizontal distance from your feet to the base of the object using a measuring tape. Record the information on the chart and label the triangle.
    - Step 5: Find the angle of elevation using the clinometer. Record the angle of elevation on the chart and label the triangle.
        - To use the Clinometer:
            - Hold the clinometer so that the rounded part is pointed down.
            - Look through the straw and spot the top of object you are measuring.
            - Allow the weighted string to hang below.
            - Hold the string against the protractor. Then look at the acute measurement on the protractor.
            - Subtract the angle measure from 90°. This measurement is the angle of elevation from your eye to top of the object.

    - Step 6: Set up and solve the trigonometric equation to find the side of the triangle that represents your object.
    - Step 7: Find *your* height and add it to the value you found in Step 6. This will be the height of your object! Record the information the chart and label the triangle.
    - Step 8: Use the Pythagorean theorem to find the distance from where you are standing to the top of the object. Hint: This distance is the hypotenuse of the triangle. Record the information on the chart and label the triangle.
    - Step 9: Repeat this process for three more objects. Example calculation of the height of a flagpole is below.

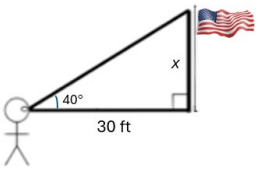
- Example calculation for the height of a flagpole. Please reference steps below.
  - Step 1: The chosen object is a flagpole.
  - Step 2: Standing position is 30 feet away from the flagpole.
  - Step 3: Draw a right triangle where:
    - The base represents the horizontal distance from your position to the base of the flagpole.
    - The height represents the unknown height of the flagpole.
    - The hypotenuse represents the line of sight from your eyes to the top of the flagpole.



  - Step 4: The measured distance from the person's feet to the base of the flagpole is 30 ft.



  - Step 5: Using the clinometer, the angle of elevation is 40°.



  - Step 6: $\tan(angle)\, x\ adjacent\ side = opposite\ side$

$$\tan(40°)\, x\ (30) = 25.173\ feet$$

  - Step 7: $x =$ height of person + height from Step 6

$$x = 5.5\ ft + 25.173$$
$$x = 30.673\ feet$$

  - Step 8: $a^2 + b^2 = c^2$

$$(30.673)^2 + (30)^2 = c^2$$
$$940.83 + 900 = c^2$$
$$1840.83 = c^2$$
$$42.9\ ft = c$$

  - Step 9: Choose other objects, such as a building, statue, or telephone pole.

Conclusion (10 minutes)

- Have the students answer the following discussion questions on their worksheet.
    - Why did you need to subtract 90 degrees from your clinometer reading to get the angle of elevation?
        - When looking through the straw straight ahead, this represents an angle of zero degrees. The string, however, will be hanging at the 90-degree measurement. This is a difference of 90 degrees.
    - Why did you need to add your height in order to get the height of the object?
        - This extra height is not accounted for in the triangle representation of your object. This additional height should be added to get a more accurate measurement of the height of the object. Technically, you should add the exact measurement from the ground to your eyes!
    - You created a simple clinometer that measures angles. Can you think of professions in which a tool like this might be helpful?
        - Answers vary. Some examples: A park ranger may complete the measurement at several different times throughout the year to measure the rate of growth of a large tree. Hikers may use this to determine the incline of a path. It could also be used in various occupations that might need a quick measurement of slope or angle of a land feature, such as a cave or a road.
- Have a whole class discussion about the reflection questions.

Differentiation

Grouping

- Each student will be required to fill out their own table, although they may check their answers with their group.
- The groups will be heterogeneous so that students can support one another's learning.

Students who are struggling

- Schedule regular check-ins with each group to assess progress and provide additional support where needed.
- Pair students who grasp concepts quickly with those who might need more time, promoting peer learning.

ELL students

- Provide the students with a translation of the terms used in the lesson.
- Allow the students to use an online translator.
- Students may draw pictures instead of written or typed answers.

Advanced students

- Pair students who grasp concepts quickly with those who might need more time, promoting peer learning.
- The students can help facilitate the experiment portion.

Assessment

Formative Assessment:

- Participation in the class discussion during the introduction (e.g., brainstorming ways to measure large objects and discussing the clinometer).
- Engagement in small group discussions during the activity (e.g., sharing ideas and checking answers with group members).
- Teacher check-ins with groups to assess progress, provide support, and ensure understanding.
- Observation of group collaboration and problem-solving throughout the activity.

Summative Assessment:

- Completion of the Trigonome-Tree Worksheet, including calculations and labeled diagrams for each object measured.
- Answers to reflection questions on the worksheet (e.g., explaining the subtraction of 90 degrees, adding height, and professions that use tools like a clinometer).
- Overall performance assessed using the rubric provided, which evaluates accuracy, completeness, and understanding of concepts.

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Trigonome-Tree Worksheet

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	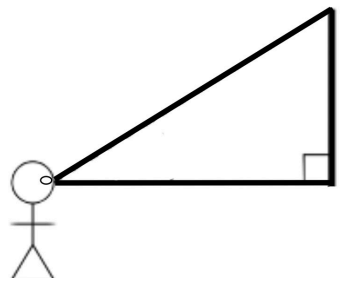					
	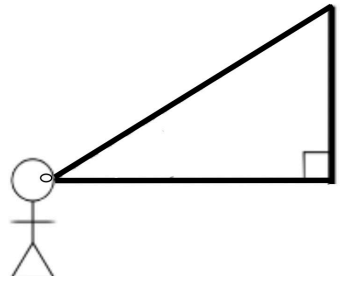					
	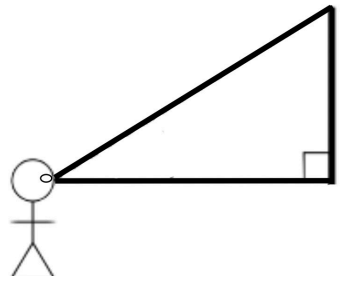					
	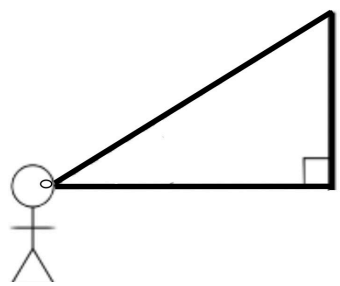					

Reflection questions:

1) Why did you need to subtract 90 degrees from your clinometer reading to get the angle of elevation?

2) Why did you need to add your height in order to get the height of the object?

3) You created a simple clinometer that measures angles. Can you think of professions in which a tool like this might be helpful?