Activity: Calculating Average Distance Biomass will Travel to a Biorefinery

Grade Level: 10th grade Geometry

Alignment to Standards:

- **G.GMD.A.2** Know and use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems.
- **G.MG.A.2** Apply geometric methods to solve real-world problems.

Lesson Objectives:

- Find a one year production density
- Find the volume of a complex solid
- Analyze units in the context of a problem
- Derive a simplified formula from known geometry formulas

Materials: None

Time Required: 15 – 20 minutes

Background Information: Bioenergy is energy derived from living matter on the surface of the earth. In the United States, we are at a historic high of renewable energy use. Bioenergy is the highest source of renewable energy. Biomass, the raw source of bioenergy, includes forests, agriculture, trash and algae. Like an oil refinery takes crude oil and converts it to a form that can be used commercially, a biorefinery takes biomass and converts it to a usable form by the consumer.

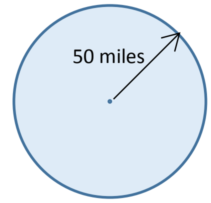
Focusing on agriculture, there are factors to consider when deciding if a biorefinery location will be viable. One consideration is how much material can be grown near the biorefinery and another is transportation costs. Transportation costs depend on the average distance material needs to travel in order to reach the biorefinery.

Below is a flow chart that shows the general steps that are taken in creating biofuel from agriculture. The following problem focuses on the two bolded sections.

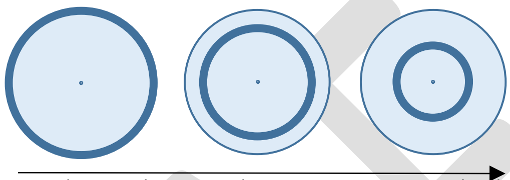
Name:____ Date:________________ Period:____________

Biofuel Application: The specifications for a planned biorefinery require 800,000 dry tons of biomass per year. The designers expect that all the biomass will come from a 50 mile radius from the biorefinery.

1) What is the biomass production density of material produced in one year, in dry tons per square miles? Round to the nearest integer.



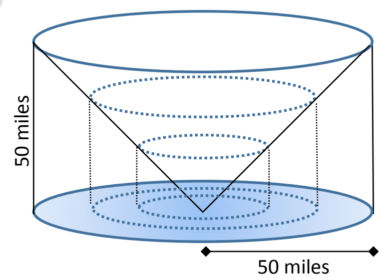
To calculate cost to transport the biomass to the biorefinery, the average distance that biomass is transported is needed. If you view crops as concentric circles, you can see that the outermost ring has the most biomass (biggest circumference). As the circles get closer to the center, the location of the biorefinery, the distance decreases, but so does the amount of biomass that needs to be transported.



As transport distance decreases, the amount to transport also decreases.

To calculate the average distance (not just half of the radius!) that biomass is transported, a 3D model is employed, and the following formula is used. Note that the height of the cylinder is equal to the radius.

$$\text{Average transport distance} = \frac{\text{Volume of cylinder - Volume of cone}}{\text{Area of base}}$$



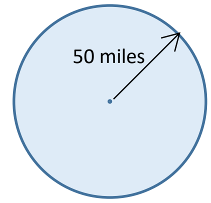
2) Calculate the average transportation distance for biomass that has been grown within 50 miles of the biorefinery. Round your answer to the nearest integer.

3) What should the units on the answer be? Do those units make sense from the given formula?

4) Starting with the formula above, rewrite and simplify to find a new formula that has the radius as the only variable.

Answer Key

Biofuel Application: The specifications for a planned biorefinery require 800,000 dry tons of biomass per year. The designers expect that all the biomass will come from a 50 mile radius from the biorefinery.



1) What is the biomass production density of material produced in one year, in dry tons per square miles? Round to the nearest integer.

$$\frac{800{,}000}{\pi (50)^2} \approx 102 \frac{\text{dry tons}}{\text{mi}^2}$$

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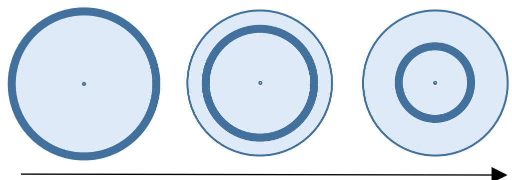

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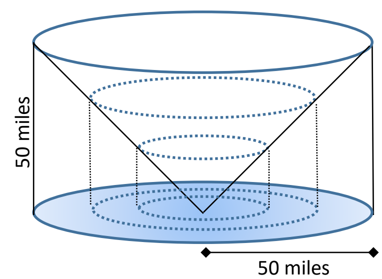

2) Calculate the average transportation distance for biomass that has been grown within 50 miles of the biorefinery. Round your answer to the nearest integer.

$$\text{Ave. distance} = \frac{\pi r^2 h - \frac{1}{3}\pi r^2 h}{\pi r^2} = \frac{\pi (50)^2(50) - \frac{1}{3}\pi (50)^2(50)}{\pi (50)^2} \approx 33$$

3) What should the units on the answer be? Do those units make sense from the given formula?

Units should be miles. Answer should contain explanation of units of volume divided by units of area resulting in a linear dimension.

4) Starting with the formula above, rewrite and simplify to find a new formula that has the radius as the only variable.

$$\text{Ave. distance} = \frac{\pi r^2 h - \frac{1}{3}\pi r^2 h}{\pi r^2}$$

(given: $h=r$)

$$= \frac{\pi r^2 \cdot r - \frac{1}{3}\pi r^2 \cdot r}{\pi r^2}$$

$$= \frac{\pi r^3 - \frac{1}{3}\pi r^3}{\pi r^3}$$

$$\text{Ave. distance} = \frac{\frac{2}{3}\pi r^3}{\pi r^2}$$

$$\boxed{\text{Ave. distance} = \frac{2}{3} r}$$