



# What Factors Correlate to Olympic Success?

Submitted by: Sarah Hampton, Algebra  
Sullins Academy, Bristol, VA

**Target Grade:** Algebra

**Time Required:** approximately eight 45-minute sessions. (\*Ideally, these would be split between math, geography, science, and technology classes.)

## Standards:

*Note: I teach in an independent school on the VA/TN state line. Therefore, I have included national standards and VA and TN state standards. These standards can be found at the end of the lesson.*

## Objectives

The learner will:

- Use the scientific method to design a way to ask and answer a relevant question.
- Brainstorm realistic factors that contribute to Olympic success.
- Search for data using credible sources.
- Locate countries on a map.
- Create bar graphs and histograms to graphically display data.
- Input data in a spreadsheet and use appropriate vocabulary to describe the data's location.
- Analyze data in a spreadsheet using graphs.
- Use linear regressions to show and quantify correlations.
- Determine the strength of the line of best fit using the correlation coefficient.
- Interpret the graphs and lines of best fit to determine which factors correlate the most to a country's average Olympic success.
- Write a mathematical argument for the factor that best correlates to Olympic success.

## Central Focus

The purpose of this cross-curricular project is for algebra students (8<sup>th</sup> to 10<sup>th</sup> grades) to generate factors that may contribute to overall Olympic success of a country, design a way to assess the correlation of those factors, execute their design, and interpret the results.



## Materials:

- World Map
- Push pins
- Computer or tablet for each student if possible
- Internet access
- Google Drive login for each student (to create a class spreadsheet using Google Sheets)
- Poster board
- Construction Paper
- Markers
- Rulers
- Yard Sticks
- Scissors
- Glue

## Hook/Opening:

Say, “After watching the last Olympics, I noticed that some countries tended to do better than other countries. Why do you think that might be?”

## Procedure:

### Day 1

1. Use the hook above to start a class discussion.
2. Show students [the activity model for scientific inquiry](#). Let them know that we have already entered the model by making an observation and defining the problem.
3. Ask students what the next step is. *They should answer “forming a question.”*
4. Have students generate a succinct question that can be answered. Scaffold as necessary.  
*Our class came up with, “What factors contribute to a country’s success in the Olympics?” Later, we discussed the difference between correlation and causation and how the more appropriate question for the math we knew how to do would have been, “What factors correlate to a country’s success in the Olympics?”*
5. Ask students what the next step in the activity model for scientific inquiry is.  
*They should answer, “Investigating the known.”*
6. Ask students to agree upon how they will measure a country’s success in the Olympics. Scaffold as necessary. If the class discussion is too overwhelming, break into groups. Support the groups who need more direct instruction.  
*Our class agreed upon total medal count. There was a good debate about this. Some students said it should be a weighted average because they thought a gold medal should be worth more than a silver, etc.*
7. Ask students where they could find information on a country’s total medal count.  
*Our class used a search engine and found this [All-time Olympic Games Medal Table](#). Note: We liked that you can sort the table by selecting the arrows at the top. We discussed how Wikipedia is not always a credible source. Afterward, I*



*discovered that the Olympics site also has this information, although it is not organized as nicely. Check out the [United States page](#) here and scroll down to see Medals at the bottom. You can keep selecting more to generate former Olympics. Alternatively, you can use [Wolfram Alpha](#) to search for medal counts.*

8. Give students time to investigate the known by locating and listing about five countries that have been very successful in the Olympics over time.

*I recommend assigning this for homework if students do not finish during class.*

## Day 2

1. Ask students what the next step in the scientific inquiry activity model is.  
*They should answer, "Articulating the expectation."*
2. Have students brainstorm what factors they think might correlate to a country's Olympic success using the list of countries that have been most successful in the past. Record class ideas or designate a student to record them. Scaffold as necessary.  
*An online thought sharing/brainstorming site could be used for collaboration also. I like [Trello](#) and [bubbl.us](#).*
3. Refine thoughts to include four or fewer main factors they want to investigate.  
*Our class reasoned that countries with more people in them had a wider talent pool to choose from. They also said that athletes in richer countries probably had the luxury of time to train and access to better training facilities. For winter Olympics, they also thought that colder countries would do better because they would have more authentic practice conditions.*
4. Ask students how the factors they generated could be measured.  
*Our class suggested population to measure how many people were in a country. They originally suggested GDP to measure how rich a country is but later changed it to per capita income after realizing that larger countries would probably have a higher GDP but might have to be distributed over more people. They also suggested that temperature was a good measure of a country's coldness. They eventually refined this to average yearly temperature.*
5. Based on their conversation, have students predict if each factor is positively or negatively correlated to Olympic success.  
*Our class predicted that population and per capita income would be positively correlated with general Olympic success and that temperature would be negatively correlated for Winter Olympic success.*
6. Ask students what the next step in the scientific inquiry activity model is.  
*They should answer, "Carrying out the study."*
7. Have students propose ideas for how we could carry out a study to determine how these factors might correlate to Olympic success.  
*Our class realized that we could collect data on a country's population, per capita income, temperature, and total medal count and try to detect trends in the data.*
8. Ask each student to pick a certain number of countries to research and place a pushpin on the world map when they have claimed it.  
*I originally asked each of my twelve students to research five countries. We later realized we could have improved our study by including more countries.*



9. Instruct each student to collect data for their countries' populations, per capita incomes, temperature, and medal counts and add them to a class spreadsheet. *There was so much productive commentary during this phase! Some students didn't know where to look so classmates shared their sources. They quickly realized that they needed to standardize their sources because data was collected at different times for different sources. In the end, they used [World Bank](#) data for population and GDP, [Lebanese Economy Forum](#) for average yearly temperature and Wikipedia for medal counts. (Later, they calculated per capita income by dividing GDP by the population.) More productive conversations followed as students wondered how to consolidate their data. After tossing around different ideas, they decided on a shared spreadsheet. Since all our students already have Google accounts for their school email addresses, I introduced them to Google Sheets. (Most were more familiar with Excel.) This is our finished [class spreadsheet](#). I suggest having students finish researching their data and adding it to the class spreadsheet for homework after session two.*

### Day 3 - 5

1. Display the class spreadsheet and make sure all data has been entered.
2. Organize the spreadsheet effectively and create any new columns you may need. Capitalize on organic teaching moments and insist on proper spreadsheet vocabulary.

*For example, the students realized that per capita income was a better measure than GDP, so this was a perfect time to teach how to use a formula in a spreadsheet. We divided the GDP column by the population column to generate the per capita income. The Lebanese Economy Forum gave the temperature in degrees Celsius but students wanted to convert that to Fahrenheit. That made for a fun algebra review and mini-assessment on how to use formulas in spreadsheets.*

3. Ask students what the next step in the scientific inquiry activity model is. *They should answer, "Examining the results."*
4. Ask students what we could do to help us detect trends in the results. *Our class remembered learning about different ways to display data and decided to make bar graphs, histograms, and scatterplots.*

5. Have students form groups and create graphical displays for the data using the poster board, markers, construction paper, rulers, yard sticks, scissors, and glue. Offer as much student choice here as possible. For example, allow students flexible grouping, alternate materials, or alternate methods of display.

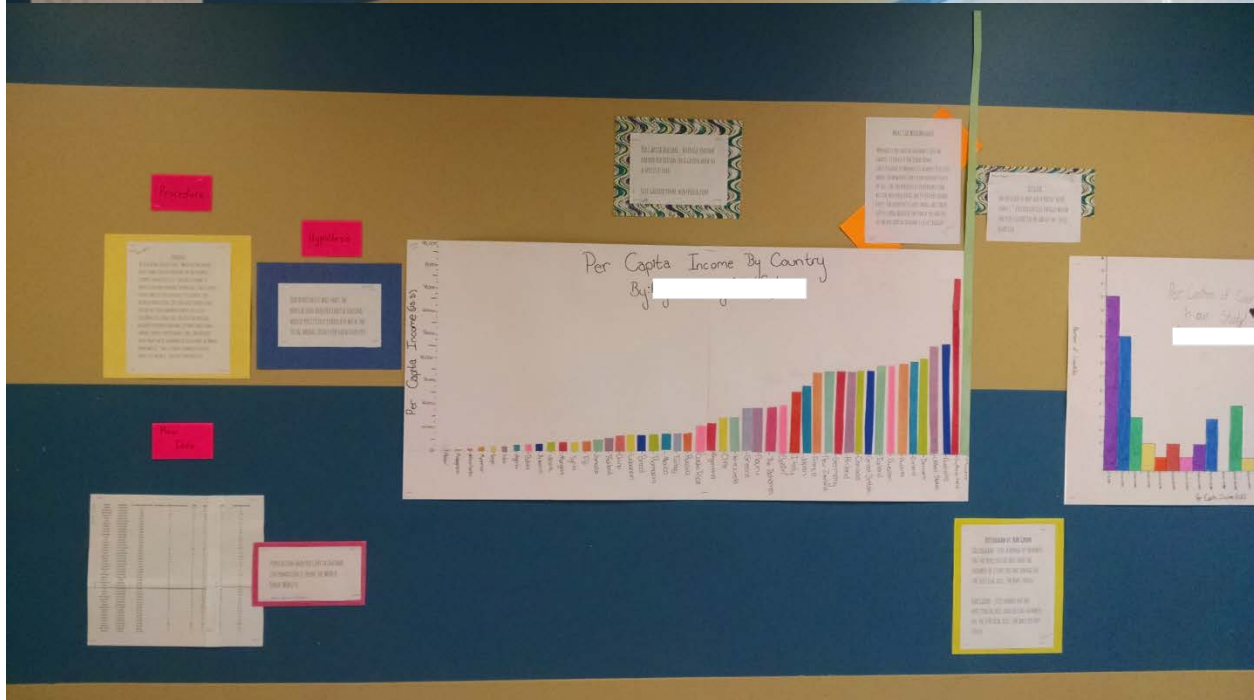
*I had three groups and assigned population to one, per capita income to another, and temperature to the third. While other groups were working on their univariate graphical displays, I worked with one group at a time on creating scatterplots from spreadsheets and on creating and interpreting lines of best fit. Each group was then assigned to map their factor against the total medal count, create the line of best fit, and add the linear regression equation and correlation coefficient.*

*Since it is difficult to do group work at home, I recommend assigning homework to practice creating and interpreting linear regressions. Use this homework*





*assignment to identify and intervene one on one with students who need extra help while other group members continue working on the graphs. These are our class results:*





## Do Population & Per Capita Income Affect Summer Olympic Performance?

Per Capita Income

Population of Countries in Our Project

8th Grade

Summer Olympic Medal Averages vs. Per Capita Income (Includes Monaco)

Summer Olympic Medal Averages vs. Country Population (Includes India and China)

Summer Olympic Medal Averages vs. Per Capita Income (Other countries - does not include Monaco)

Summer Olympic Medal Averages vs. Country Population (Population outliers removed - does not include India and China)

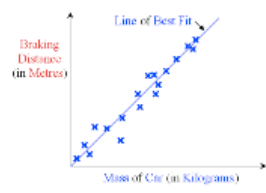


Day 6

1. Ask students what the next step in the scientific inquiry activity model is. *They should answer, "Reflecting on the findings."*
2. Ask the students to discuss what surprised them as they were creating the graphs. Capitalize on just in time learning moments. *For example, students noticed that Iraq and Canada had similar populations on the bar graph but also noticed the significant difference in area on the world map. This naturally led to a discussion on population density. Students also noticed that some of the values were literally off the charts--like the per capita income of Monaco. This presented the perfect opportunity to discuss the term outlier.*
3. When a student's observation or curiosity leads them to ask another question, encourage them to do an extension. Offer student choice on how this will be displayed. Perhaps the poster is actually a QR code that leads to a video explanation instead of a written explanation. *I required every student to choose at least one extension on a topic of interest. For example, the student who spotted the population similarity but area difference of Iraq and Canada created a vocabulary poster on population density to be displayed with the project. The student who noticed that Monaco was off the charts created a vocabulary poster on outlier. The following is an example of a student-created poster explaining lines of best fit. He picked this because he didn't think most people who stopped to look at our project would know what they were.*

LINE OF BEST FIT

THE "LINE OF BEST FIT" MAY ALSO BE CALLED A "TREND" LINE OR A "LINEAR REGRESSION." A LINE OF BEST FIT IS A LINE THAT BEST REPRESENTS THE DATA ON A SCATTER PLOT. THIS LINE MAY PASS THROUGH SOME OF THE POINTS, NONE OF THE POINTS, OR ALL OF THE POINTS.



EXAMPLE FROM:

[HTTPS://ANCASTERMATH.WIKISPACES.COM/STATISTICS+](https://ancastermath.wikispaces.com/Statistics+)

2



### Day 7

1. Have students look beyond the appearance of the data to number analysis. This should include a thorough analysis of the linear regression equations and their correlation coefficients.
2. After the class discussion, instruct each student to write a compelling mathematical argument that accurately interprets the data.  
*I recommend that students complete their arguments for homework.*

### Day 8

1. Have volunteers read their arguments to the class. Have the class constructively comment on the strengths and weaknesses of the argument.
2. Have students use their arguments to predict which countries will do best in the 2018 Olympics and add their prediction to their arguments.
3. Ask students what the next step in the scientific inquiry activity model is.  
*They should answer, "Communicating with others."*
4. Have students meaningfully display all components of their projects in a place of prominence in the school or greater community.
5. After the 2018 Olympics, revisit the predictions and discuss any discrepancies or other observations.

### **Note:**

I tried to include reflections within the procedure to help the next teacher more effectively use the lesson plan. I also wrote some further reflections on the project our class originally completed on [CIRCL Educators](#).





## Assessment of Student Performance:

### Summative Assessment

- Class created spreadsheet
- World map with push pins students used to locate their assigned countries
- Student created bar graph or histogram
- Student created scatterplots with lines of best fit
- Student created definitions for just-in-time vocabulary and/or observations
- Student written mathematical argument for the factor that most contributes to a country's Olympic success
- Student prediction of countries who will do best in the 2018 Winter Olympics

### Formative Assessment

- Class brainstorming discussion to open the project.
- Have students lead the discussion using the scientific method to ask and answer a relevant question.
- Informal Q&A during direct instruction on how to locate relevant data for the generated factors.
- Teacher spot checks spreadsheet for each student's countries to ensure accurate data, appropriate source, and location of information in the spreadsheet.
- Teacher leads a discussion on the importance of standardizing the data's recorded time and source if necessary. Students revise data if necessary.
- Teacher praises and/or corrects proper spreadsheet vocabulary during the project.
- Teacher monitors progress of the bar graph or histogram and redirects the group as necessary.
- Teacher uses direct instruction on scatterplots and lines of best fit and uses informal Q&A and homework to ensure student understanding.
- Teacher listens for student curiosity and observations and prompts the student to research and report on relevant vocabulary or observations. Teacher also encourages students to research and report more about countries as they generate new potentially relevant questions.
- Teacher pauses formal project activity and initiates direct instruction when just in time learning is needed. Teacher poses meaningful questions to informally assess student understanding. Students participate in class conversation, making meaning of what they learn together.
- Practice problems and/or homework problems will be assigned and checked when teacher notices students need practice on specific concepts. For example, practice with creating and interpreting lines of best fit on simpler data will almost certainly be beneficial.



**Summative Assessment Rubric for Olympics Project**

Total Score: \_\_\_\_\_/18

| <b>Category</b>        | <b>0</b>                                                                                                                          | <b>1</b>                                                                                                                                                                                                              | <b>2</b>                                                                                                                                                                                                                 | <b>3</b>                                                                                                                                                                                                                         |
|------------------------|-----------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| World Map              | Student does not locate his or her assigned countries on map with push pins                                                       | Student does not locate half or more of his or her countries accurately on the world map with push pins                                                                                                               | Student only locates over half but not all his or her countries accurately on the world map with push pins                                                                                                               | Student accurately locates all his or her assigned countries on the world map with push pins                                                                                                                                     |
| Spreadsheet            | Student does not attempt to input data into the class spreadsheet about his or her countries                                      | Student does not input accurate data into the class spreadsheet for his or her countries                                                                                                                              | Student only inputs accurate data into the class spreadsheet for some of his or her countries                                                                                                                            | Student inputs accurate data into the class spreadsheet for all his or her assigned countries                                                                                                                                    |
| Bar Graph or Histogram | The group does not present a bar graph or histogram                                                                               | The group presents a bar graph or histogram, but some of the information is inaccurate                                                                                                                                | The group presents an accurate bar graph or histogram but it is not easy to read                                                                                                                                         | The group presents an accurate bar or histogram that is easy to read                                                                                                                                                             |
| Scatterplot            | The group does not present a scatterplot<br><b>OR</b><br>The group presents a scatterplot, but it contains inaccurate information | The group presents a scatterplot with accurate information, but it is missing more than three of the required components: title, axis labels, scale, line of best fit, equation for line of best fit, and correlation | The group presents a scatterplot with accurate information, but it is missing one, two, or three of the required components: title, axis labels, scale, line of best fit, equation for line of best fit, and correlation | The group presents a scatterplot with accurate information that has all of the required components: title, axis labels, scale, line of best fit, equation for line of best fit, and correlation coefficient for line of best fit |



|                                        |                                                                                                   | coefficient for<br>line of best fit                                                                                  | coefficient for<br>line of best fit                                                                                                                                                                                         |                                                                                                                                                                                         |
|----------------------------------------|---------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Definition or<br>Observation<br>Poster | The student<br>does not<br>present a<br>poster with a<br>relevant<br>definition or<br>observation | The student<br>presents a<br>poster, but it is<br>not relevant to<br>the project                                     | The student<br>presents a<br>relevant<br>definition or<br>observation,<br>but some of<br>the content is<br>inaccurate                                                                                                       | The student<br>presents a<br>relevant and<br>accurate<br>definition or<br>observation                                                                                                   |
| Mathematical<br>Argument               | The student<br>does not<br>present a<br>written<br>argument or<br>prediction                      | The student<br>presents a<br>written<br>argument with<br>prediction, but<br>it is not<br>supported by<br>mathematics | The student<br>presents a<br>written<br>mathematical<br>argument but<br>misinterprets<br>the<br>mathematics<br><b>OR</b><br>The student<br>does not<br>construct<br>compelling<br>reasoning for<br>their 2018<br>prediction | The student<br>presents a<br>written,<br>accurate, and<br>compelling<br>mathematical<br>argument that<br>interprets past<br>data and<br>predicts future<br>data in the<br>2018 Olympics |



**Resources:**

Harwood, W. (2003, December 23). *An Activity Model for Scientific Inquiry*. Retrieved from: <http://www.nsta.org/publications/news/story.aspx?id=48890>

(2018, January 12). *All-time Olympic Games Medal Table*. Retrieved from: [https://en.wikipedia.org/wiki/All-time\\_Olympic\\_Games\\_medal\\_table](https://en.wikipedia.org/wiki/All-time_Olympic_Games_medal_table)

(2017). *United States of America Medals*. Retrieved from: <https://www.olympic.org/united-states-of-america>

(2018). Wolfram|Alpha. Retrieved from: <https://www.wolframalpha.com/input/?i=how+many+olympic+medals+did+US+win+in+2012>

(2018). Trello. Retrieved from: <https://trello.com/>

(2018). Bubbl. Retrieved from: <https://bubbl.us/>

(2017). World Bank Open Data. Retrieved from: <https://data.worldbank.org/>

Lebanese Economy Forum. *Average yearly temperature (1961-1990, Celsius) - by country*. Retrieved from: <http://lebanese-economy-forum.com/wdi-gdf-advanced-data-display/show/EN-CLC-AVRT-C/>

(2016). *2016 Statistics Project Data*. Retrieved from: [https://docs.google.com/spreadsheets/d/1ZPQ0HGx\\_ONyPrREW-Q-jk3SSA7WAekZnojzaVHXedS0/edit?usp=sharing](https://docs.google.com/spreadsheets/d/1ZPQ0HGx_ONyPrREW-Q-jk3SSA7WAekZnojzaVHXedS0/edit?usp=sharing)

Hampton, S. (2017, February 16). *Favorite Tech Tools Series: Google Drive*. Retrieved from: <http://www.circleducators.org/blog/favorite-tech-toolspost-1>





## Standards:

### NCTM

#### Principles of Learning as listed in the 2014 Principles of Action

- Engage with challenging tasks that involve active meaning making and support meaningful learning.
- Acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations.
- Construct knowledge socially, through discourse, activity, and interaction related to meaningful problems.

#### Mathematics Teaching Principles as listed in the 2014 Principles of Action

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Facilitate meaningful mathematical discourse.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

#### Data Analysis Standards

Instructional programs from prekindergarten through grade 12 should enable each and every student to—

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data

### Next Gen Science Standards

#### Dimension 3

The continuing expansion of scientific knowledge makes it impossible to teach all the ideas related to a given discipline in exhaustive detail during the K-12 years. But given the cornucopia of information available today virtually at a touch—people live, after all, in an information age—an important role of science education is not to teach “all the facts” but rather to prepare students with sufficient core knowledge so that they can later acquire additional information on their own. An education focused on a limited set of ideas and practices in science and engineering should enable students to evaluate and select reliable sources of scientific information, and allow them to continue their development well beyond their K-12 school years as science learners, users of scientific knowledge, and perhaps also as producers of such knowledge.

Specifically, a core idea for K-12 science instruction should:



1. Have broad importance across multiple sciences or engineering disciplines or be a key organizing principle of a single discipline.
2. Provide a key tool for understanding or investigating more complex ideas and solving problems.
3. Relate to the interests and life experiences of students or be connected to societal or personal concerns that require scientific or technological knowledge.

### **Common Core State Standards for Mathematics** Standards for Mathematical Practice

- CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
- CCSS.Math.Practice.MP2 Reason abstractly and quantitatively
- CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
- CCSS.Math.Practice.MP4 Model with mathematics.
- CCSS.Math.Practice.MP% Use appropriate tools strategically.

### High School: Statistics & Probability » Interpreting Categorical & Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables

- CCSS.Math.Content.HSS.ID. B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
- CCSS.Math.Content.HSS.ID. B.6.a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
- CCSS.Math.Content.HSS.ID. B.6.c Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

- CCSS.Math.Content.HSS.ID. C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

### **Common Core State Standards K-12 Technology Skills Scope and Sequence**

- Demonstrate an understanding of the spreadsheet as a tool to record, organize and graph information.
- Identify and explain terms and concepts related to spreadsheets (i.e. cell, column, row, values, labels, chart graph)
- Enter/edit data in spreadsheets and perform calculations using formulas
- Use spreadsheets and other applications to make predictions, solve problems and draw conclusions.
- Use spreadsheets to calculate, graph, organize, and present data in a variety of real-world settings and choose the most appropriate type to represent given data



- Demonstrate the ability to use technology for research, critical thinking, decision making, communication, collaboration, creativity, and innovation. Research (Gather and Using Information)

## **Virginia Standards:**

### **VA Algebra SOL**

#### Statistics

- A.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of linear and quadratic functions.

### **VA World Geography SOL**

- WG.1 The student will use maps, globes, satellite images, photographs, or diagrams to
  - b) apply the concepts of location, scale, map projection, or orientation;
  - c) develop and refine mental maps of world regions;
- WG.5 The student will compare and contrast the distribution, growth rates, and characteristics of human population in terms of settlement patterns and the location of natural and capital resources.

### **VA Science SOL Goals**

3. Investigate phenomena, using technology.
4. Apply scientific concepts, skills, and processes to everyday experiences.
6. Make informed decisions regarding contemporary issues, taking into account the validation from scientific data and the use of scientific reasoning and logic.
7. Develop scientific dispositions and habits of mind including: curiosity, demand for verification, respect for logic and rational thinking, patience and persistence.

### **VA English SOL**

#### Research

- 9.8 The student will use print, electronic databases, online resources, and other media to access information to create a research product.
  - a) Use technology as a tool for research to organize, evaluate, and communicate information.
  - b) Narrow the focus of a search.
  - c) Find, evaluate, and select appropriate sources to access information and answer questions.



## Tennessee Standards:

### TNReady Literacy Skills for Mathematical Proficiency

- Discuss and articulate mathematical ideas.<sup>[1]</sup><sub>[SEP]</sub>
- Write mathematical arguments.

### TN Integrated Math 1 Standards

#### Geometry and Interpreting Data

- M1.S.ID.B - Summarize, represent, and interpret data on two categorical and quantitative variables.
- M1.S.ID.C - Interpret linear models.

### TN World Geography Standards

#### The Geographer's Toolkit

The student will use maps, globes, satellite images, photographs, or diagrams for geographic purposes.

- WG.3 - Obtain geographical information about the world's countries, cities, and environments.
- WG.4 - Apply the concepts of location, scale, map projection, or orientation.
- WG.5 - Develop and refine mental maps of world regions.

#### The Relevance of Geography

- WG.32 - Use geographic knowledge, skills, and perspectives to analyze problems and make decisions
- WG.33 - Relate current events to the physical and human characteristics of places and regions.

### TN Science Declarations

- Make pertinent connections among scientific concepts, associated mathematical principles, and skillful applications of reading, writing, listening, and speaking
- Think critically and logically to analyze and interpret data, draw conclusions, and develop explanations that are based on evidence and are free from bias;
- Communicate and defend results through multiple modes of representation (e.g., oral, mathematical, pictorial, graphic, and textual models)
- Integrate science, mathematics, technology, and engineering design to solve problems and guide everyday decisions
- Locate, evaluate, and apply reliable sources of scientific and technological information

### TN Writing Standards Cornerstones/Common Core Writing Standards





- Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
- Conduct short as well as more sustained research projects based on focus questions, demonstrating new understanding of the subject under investigation
  - 9-10.W.RBPK.7 Conduct and write short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem by narrowing or broadening the inquiry when appropriate, synthesizing multiple sources on the subject, and demonstrating a new understanding of the subject under investigation.