



# Designing Product Packaging: The Criteria of Volume and Constraint of Surface Area

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**Target Grade:** 8th Grade

**Time Required:** 10 Sessions: 45 Minutes per Session

**Standards:**

## **NCTM**

### Geometry Standards

Grades 6–8 Expectations: In grades 6–8 each and every student should–

- draw geometric objects with specified properties, such as side lengths or angle measures;
- use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- use geometric models to represent and explain numerical and algebraic relationships;
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

## **Next Gen Science Standards**

### Dimension 3

Specifically, a core idea for K-12 science instruction should:

- Have broad importance across multiple sciences or engineering disciplines or be a key organizing principle of a single discipline.
- Provide a key tool for understanding or investigating more complex ideas and solving problems.
- Relate to the interests and life experiences of students or be connected to societal or personal concerns that require scientific or technological knowledge.

### MS. Engineering Design

Students who demonstrate understanding can:

- MS-ETS1-2. Evaluate competing design solutions using a systematic process to determine how they meet the criteria and constraints of the problem.



## **ITEEA Standards**

Students will develop an understanding of Design. This includes knowing about:

### The attributes of design

In order to comprehend the attributes of design, students in Grades 6-8 should learn that

- Design is a creative planning process that leads to useful products and systems. The design process typically occurs in teams whose members contribute different kinds of ideas and expertise. Sometimes a design is for a physical object such as a house, bridge, or appliance and sometimes it is for a non-physical thing, such as software.
- There is no perfect design. All designs can be improved. The best designs optimize the desired qualities— safety, reliability, economy, and efficiency — within the given constraints. All designs build on the creative ideas of others.
- Requirements for a design are made up of criteria and constraints.
  - Criteria identify the desired elements and features of a product or system and usually relate to their purpose or function
  - Constraints, such as size and cost, establish the limits on a design.

### Engineering design

In order to comprehend engineering design, students in Grades 6-8 should learn that:

- Brainstorming is a group problem-solving design process in which each person in the group presents his or her ideas in an open forum. In this process, no person is allowed to criticize anyone else's ideas regardless of how inane they may seem. After all of the ideas are recorded, the group selects the best ones, and then further develops them.
- Modeling, testing, evaluating, and modifying are used to transform ideas into practical solutions. Historically, this process has centered on creating and testing physical models. Models are especially important for the design of large items, such as cars, spacecraft, and airplanes because it is cheaper to analyze a model before the final products and systems are actually made. Evaluation is used to determine how well the designs meet the established criteria and to provide direction for refinement. Evaluation procedures range from visually inspecting to actually operating and testing products and systems.

Students will develop Abilities for a Technological World. This includes becoming able to:

### Apply the design process

As part of learning how to apply design processes, students in Grades 6-8 should be able to:



- Specify criteria and constraints for the design include function, size, and materials, while examples of constraints are costs, time, and user requirements. Explore various processes and resources and select and use the most appropriate ones. These processes and resources should be based on the criteria and constraints that were previously identified and specified.
- Make two-dimensional and three-dimensional representations of the designed solution. Two-dimensional examples include sketches, drawings, and computer-assisted designs (CAD). A model can take many forms, including graphic, mathematical, and physical.
- Test and evaluate the design in relation to pre-established requirements, such as criteria and constraints, and refine as needed. Testing and evaluation determine if the proposed solution is appropriate for the problem. Based on the results of the tests and evaluation, students should improve the design solution. Problem-solving strategies involve applying prior knowledge, asking questions, and trying ideas.
- Make a product or system and document the solution. Group process skills should be used, such as working with others in a cooperative team approach and engaging in appropriate quality and safety practices. Students should be encouraged to use design portfolios, journals, drawings, sketches, or schematics to document their ideas, processes, and results. There are many additional ways to communicate the results of the design process to others, such as a World Wide Web page or a model of a product or system.

## **Common Core State Standards for Mathematics**

### Standards for Mathematical Practice

- CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
- CCSS.Math.Practice.MP2 Reason abstractly and quantitatively
- CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
- CCSS.Math.Practice.MP4 Model with mathematics.

### **Grade 8: Geometry**

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

- CCSS.Math.Content.8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## **Common Core State Standards for Grade 8 English Language Arts in Speaking and Listening**

### **Comprehension and Collaboration**



- CCSS.ELA-Literacy.SL.8.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clearly.
- CCSS.ELA-Literacy.SL.8.1.b Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed.
- CCSS.ELA-Literacy.SL.8.1.c Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas.
- CCSS.ELA-Literacy.SL.8.1.d Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented.
- CCSS.ELA-Literacy.SL.8.3 Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced.

### **Presentation of Knowledge and Ideas**

- CCSS.ELA-Literacy.SL.8.4 Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation.
- CCSS.ELA-Literacy.SL.8.6 Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 8 Language standards 1 and 3 here for specific expectations.)

### **Common Core State Standards for Grade 8 English in Language**

- CCSS.ELA-Literacy.L.8.1 Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
- CCSS.ELA-Literacy.L.8.3 Use knowledge of language and its conventions when writing, speaking, reading, or listening.
- CCSS.ELA-Literacy.L.8.6 Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.

### **Lesson Objectives:**

Students will:

- Identify criteria and constraints of a design task.
- Brainstorm designs that meet criteria and constraints.
- Draw 2d representations of 3d objects on paper with defining characteristics labeled.



- Calculate the surface area and volume of a prism, pyramid, cylinder, cone, and sphere.
- Evaluate competing design solutions using a systematic process to determine how they meet the criteria and constraints of the problem.
- Model 3d objects with online CAD software.
- Build a physical model of an optimal 3d design.
- Measure dimensions with a metric ruler.
- Develop a mathematical defense using evidence and logic.
- Articulate a mathematical defense using proper vocabulary, command of the English language, appropriate verbal and nonverbal presentation skills, and tone appropriate to the audience, topic, and purpose.
- Respond to questions and comments of audience.
- Critique the arguments of others.
- Work on a team respectfully and productively.

### Central Focus:

The purpose of this cross-curricular project is for 8<sup>th</sup> grade students a.) to design and model a product container that minimizes surface area while maximizing volume using one piece of poster board and b.) to develop and deliver a pitch that mathematically defends their design.

### Background Information:

Some background information that students will need to have good skills in English, science and math leading up to the 8th grade standards. Some example is to be able to respond to questions thoroughly, in good, complete sentences. They will also need to know how to use the metric system and be able to work effectively in groups.

### Materials

- Metric rulers
- Meter sticks
- Metric tape measures
- Compasses (or thumbtacks, string, and pencils)
- 1 piece of poster board for each student
  - They will work in groups, but this will allow each student to measure and play as desired and will also provide additional materials for mistakes.
- Scissors
- Tape
- Pencils
- Student activity sheet and additional paper or notebook for recording notes, sketches, and calculations
- Device with internet access for each group (ideally for each student)
- Calculator for each student (may use [Desmos](#) or online equivalent)
- [Student activity sheet](#), [activity sheet key](#), and [scoring rubric](#)

### Instruction

#### Session 1



1. Hook: Lead a discussion around this question: Why do you think companies sometimes change their packaging when they aren't changing the product itself?
2. Hand out the student activity sheets and have students record the most compelling answer(s) from the hook discussion.
3. Have students read and discuss the [6 Reasons to Consider a Product Packaging Redesign](#) article in groups of 3-4.
4. Read the scenario from the activity sheet: "An oatmeal company has recently seen an increase in the cost to produce their packaging container. They have hired you as a consultant. Your job is to evaluate the dimensions and shapes of a possible new container that will hold the most oatmeal while minimizing the cost associated with the packaging material and then model the different dimensions and shapes and build the best design. Each container can only be made with the amount of material equivalent to one poster board."
5. Discuss the task as a whole group to ensure task comprehension and allow time for clarifying questions. Ask students to identify which reason of the six they read about explains why this company wants to change their packaging design and record it on the activity sheet.
6. Give students more specific expectations for the project . "Your work must include:
  - a. 2d representations of your 3d figures with defining characteristics labeled and corresponding surface areas and volumes calculated. You should have five representations: one prism, one pyramid, one cylinder, one cone, and one sphere. Try to maximize the volume for each of these shapes without using more than one posterboard's worth of material. (Play around with the [Shodor online interactive](#) or the [Learn Alberta interactive](#) to get a feel for how changing dimensions affects surface area and volume.)
  - b. A [Tinkercad](#) representation of your prism, pyramid, cylinder, cone, and sphere.
  - c. A physical model of your most effective design.
  - d. An oral presentation that mathematically and verbally persuades the audience of your design."
7. Have students break into their groups and brainstorm the criteria and constraints of the task. Remind them to document their brainstorming either on paper or using brainstorming software such as [Trello](#) or [Bubbl.us](#). Students should recognize the explicit constraint to save material cost but may need scaffolding to connect that to surface area, specifically the surface area equivalent to or less than that of one piece of posterboard. Likewise, students should recognize the explicit criteria of holding the most oatmeal but may need scaffolding to connect that to volume. If the students did not generate implicit constraints (the shape needs to be free standing; ease of use should be considered, etc.), then do not direct their thinking yet. These implicit constraints should come to light as students evaluate competing design solutions and build prototypes. Have students complete the criteria and constraints portion of the activity sheet. (Note: Even though students are working in groups, every student must complete their own activity sheet so they can individually demonstrate their competencies in each standard) While students are working groups, look for and document evidence of brainstorming and member participation.



### Sessions 2, 3, and 4

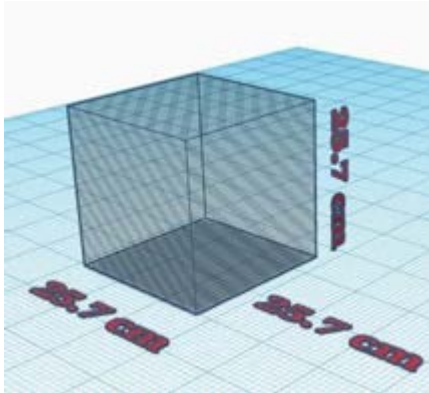
1. Instruct students to meet back in their groups and begin working on their 2d representations and calculations. This task is inherently “low floor, high ceiling.” Students with a sophisticated understanding of algebra could go as far as solving for the dimensions for the rectangular prism or sphere. (To solve for the optimal condition of the pyramid, cylinder, or cone would even require calculus!) At the same time, students with a limited understanding of order of operations can still succeed in the task using a guess and check method. As you monitor and chat with each group, encourage those using the guess and check method to streamline their process by investigating the [Shodor online interactive](#) or the [Learn Alberta interactive](#) to get a feel for how changing dimensions affects surface area and volume. Allow students to use online surface area and volume calculators while they identify optimal conditions. However, require them to show each step from the formula to final surface area and volume on the activity sheet for the optimal design for each shape. The goal of this task is for students to make sense of the formulas as they anchor their calculations in a real world context. At this level, it is more important for students to mathematically defend their chosen design than to reach the absolute optimal conditions. (Note: This is also a great opportunity to discuss why rounding down is sometimes better than traditional rounding so they do not exceed the area of the poster board. In addition, it is important for students to recognize that, although you can theoretically solve for optimal conditions to the thousandths of a centimeter and beyond, using a metric ruler, humans cannot accurately measure beyond millimeters. Groups should allow for that buffer in instrumental and human imperfection by giving answers no more precise than millimeters.) This portion of the project will take the most time and represents about half of the learning goals.
2. **Differentiation Strategies:** If students are having trouble remembering the formulas, this [color wheel foldable](#) looks like a great resource! If students need more practice calculating surface and volume, then direct them to more practice on their trouble spots using this [Khan Academy page](#). If students need an extension, see if they can extend their understanding of surface area and volume to the size of cells. [The Sizes of Organisms: The Surface Area to Volume Ratio](#) site and the [Exploring Life at Bio dot Edu](#) are two great places to start. Another extension idea is to design a logo for the product packaging. Check in with each group frequently throughout the project. Have them score themselves on the rubric for their current progress intermittently. (This holds them accountable for reading the rubric, helps them become more independent, and helps improve metacognition.) Help them identify any discrepancies between their scores and the score you would have given for their completed parts to date. Coach them on time management if necessary. Support any strategy the group chooses for division of labor as long as each student can demonstrate understanding of the surface area and volume calculations for each shape. Intervene as necessary with direct instruction.

### Session 5

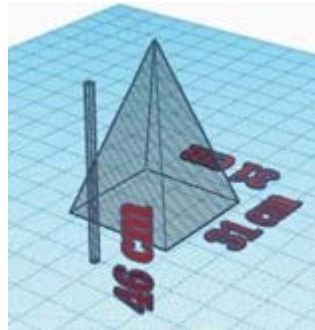


By this point, all groups should be finished with their optimal designs for each of the five shapes. Let them know that they will be using an online CAD software called [Tinkercad](#) to make models of those designs. Remind them that they must label their dimensions and may even want to add surface areas and volumes. Also remind them that, in order to receive full credit, the dimensions must be realistic. Each student should make at least one of the shapes using Tinkercad.

If the group finishes this early, they may begin on their physical model.



Ex:



Ex:

### Session 6

Instruct students to finish their Tinkercad models if necessary. Once they have completed those, instruct them to start building their physical model using the poster board. Remind them that they only have a few pieces per group so measure twice and cut once! By the end of this session, students should be done with the modeling portion and ready to move onto their defense articulation/pitch.

### Session 7

Today, have the groups discuss their strategy for the defense/pitch and write an outline. They should include the elements of a good pitch as well as supporting mathematical evidence. Use [this Forbes site](#) for pitch elements.

### Session 8

Students should practice their pitch today. It would be helpful if each group was able to do a mock run with you. First, ask the group what they think are the strengths and weaknesses of the pitch using the rubric. Then, give your own feedback. Also ask them to anticipate what comments or questions the audience might have to prepare a response. If groups are prepared for tomorrow's pitch and time remains, have them design a logo for their new package.





### Sessions 9 and session 10

To conclude the project, it's time to pitch! While groups take turns delivering their defenses/pitches, other students take notes as they critique their logic. At the end of each pitch, there should be a Q&A session. Score each group against the rubric as they offer critiques and respond to the audience. Toward the end of session 10, have a class discussion recapping important elements of the project such as why companies might redesign packaging, what makes something a criteria or constraint, what process they used to determine the optimal design, how they employed the engineering design process, etc. Also, highlight the strengths of each group's design. At the end of class, have students display their Tinkercad models and 3d models in a prominent location for the school community to see. The teacher should add an informational poster about the project.

### **Differentiation**

This lesson has a lot of differentiation because it reaches most types of learners. It is important that the teacher uses differentiated instruction as they go through each step. It is always good to have something that is hands-on for your students who need to do it to understand it, which is exactly what this lesson does. Students get to work in groups and have worksheets to do on their own to help them use each other to think together, but still get to practice on their own.

### **Assessment**

#### Formative Assessment

- Class brainstorming discussion on product packaging redesign to open the project.
- Teacher spot checks activity sheet and intervenes as necessary.
- Groups score themselves on the rubric for their current progress intermittently. (This holds them accountable for reading the rubric, helps them become more independent, and helps improve metacognition.)
- Teacher helps group identify any discrepancies between their scores and teacher score.
- Teacher looks for evidence of member contribution and encourages more if necessary.
- Informal Q&A and homework to ensure student understanding.
- Teacher poses meaningful questions to informally assess student understanding.
- Students participate in group and class conversations throughout, making meaning of what they learn together.
- Practice problems and/or homework problems will be assigned and checked when teacher notices students need practice on specific concepts. For example, practice with



calculating surface area and volume without access to a computer will almost certainly be needed.

## Resources:

(2018). Desmos. Retrieved from: <https://www.desmos.com/calculator>

(2018). Google Search, "Same Look New Product." Retrieved from:

[https://www.google.com/search?q=new+look+same+product&client=firefox-b-1-ab&tbm=isch&tb\\_o=u&source=univ&sa=X&ved=0ahUKEwiyvKTcsdnaAhXJg-AKHb29B14QsAQILQ&biw=1427&b\\_ih=731](https://www.google.com/search?q=new+look+same+product&client=firefox-b-1-ab&tbm=isch&tb_o=u&source=univ&sa=X&ved=0ahUKEwiyvKTcsdnaAhXJg-AKHb29B14QsAQILQ&biw=1427&b_ih=731)

Kevin. (2017, January 4). *Six Reasons to Consider a Product Packaging Redesign*. Retrieved from: <https://www.pruntrunner.com/blog/product-packaging-redesign/>

Shodor. (2018). *Surface Area and Volume Rectangular Prism Interactive*. Retrieved from: <http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/>

Learn Alberta. (2018). *Surface Area and Volume Cylinder Interactive*. Retrieved from: [http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHA P&ID2=AB.M ATH.JR.SHAP.SURF&lesson=html/video\\_interactives/areavolume/areaVolumeInteractive.html](http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHA P&ID2=AB.M ATH.JR.SHAP.SURF&lesson=html/video_interactives/areavolume/areaVolumeInteractive.html)

(2018). Tinkercad. Retrieved from: <https://www.tinkercad.com/>

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Explore Math in Demand. (2017). *Volume and Surface Area Wheel Foldable*. Retrieved from: <https://www.exploremathindemand.com/store/p5/volumeandsurfaceareawheelfoldable>

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M. Beals, L. Gross, S. Harrell. (2000). *Sizes of Organisms: The Surface Area: Volume Ratio*. Retrieved from: [http://www.tiem.utk.edu/~gross/bioed/bealsmodules/area\\_volume.html](http://www.tiem.utk.edu/~gross/bioed/bealsmodules/area_volume.html)

John Blamire. (2001). *The World of Cells: The Problem of Size*. Retrieved from: [http://www.brooklyn.cuny.edu/bc/ahp/LAD/C5/C5\\_ProbSize.html](http://www.brooklyn.cuny.edu/bc/ahp/LAD/C5/C5_ProbSize.html)

Carmine Gallo. (July 18, 2016). *The 3-Part Formula for a Winning Sales Pitch*. Retrieved from: <https://www.forbes.com/sites/carminegallo/2016/07/18/the-3-part-formula-for-a-winning-sales-pitch/#4e5aa9445d13>

## Designing Product Packaging: The Criteria of Volume and Constraint of Surface Area

### Student Activity Sheet

1. Why do you think companies sometimes change their packaging when they aren't changing the product itself?

#### The Task

An oatmeal company has recently seen an increase in the cost to produce their packaging container. They have hired you as a consultant. Your job is to evaluate the dimensions and shapes of a possible new container that will hold the most oatmeal while minimizing the cost associated with the packaging material and then model the different dimensions and shapes and build the best design. Each container can only be made with the amount of material equivalent to one poster board. "Your work must include:

- a. 2d representations of your 3d figures with defining characteristics labeled and corresponding surface areas and volumes calculated. You should have five representations: one prism, one pyramid, one cylinder, one cone, and one sphere. Try to maximize the volume for each of these shapes without using more than one posterboard's worth of material. (Play around with the [Shodor online interactive](#) or the [Learn Alberta interactive](#) to get a feel for how changing dimensions affects surface area and volume.)
- b. A [Tinkercad](#) representation of your prism, pyramid, cylinder, cone, and sphere.
- c. A physical model of your most effective design.
- d. An oral presentation that mathematically and verbally persuades the audience of your design."

2. Which of the six reasons you read about in the article explains why this company wants to change their packaging design?

3. What are the criteria and constraints of the task?

**criteria**

**constraints**

4. On the activity sheet, draw a 2d representation of your optimal design for each shape with defining characteristics labeled. Then show all work for the surface area and volume calculation for each.

**Rectangular Prism**

**Square Pyramid**

**Cylinder**

**Cone**

**Sphere**

5. Now you will be using [Tinkercad](#) to make 2d models of your shapes. Each student needs to make at least one. When you are done, either export your shapes to me, or show me in person so I can initial beside each shape.

**Prism**

**Pyramid**

**Cylinder**

**Cone**

**Sphere**

6. Physical Model - You will turn in your physical model with your activity sheets.

7. Write an outline for your defense articulation/pitch. You should include the elements of a good pitch as well as supporting mathematical evidence. Use [this Forbes site](#) for pitch elements. You will be scored on sound evidence and logic, proper vocabulary, command of the English language, appropriate verbal and nonverbal presentation skills, and tone appropriate to the audience, topic, and purpose.

8. Critique the reasoning of each group. Jot down notes of comments you want to make or questions you want to ask them.

**Group 1**

**Group 2**

**Group 3**

**Group 4**

**Group 5**

**Group 6**

**Group 7**



**Group 8**

**Group 9**

**Group 10**

This was a challenging task. Congratulations on completing it! I'm proud of you!



## Designing Product Packaging: The Criteria of Volume and Constraint of Surface Area

### Student Activity Sheet

1. Why do you think companies sometimes change their packaging when they aren't changing the product itself?

According to [Printrunner's blog](#), there are six major reasons:

1. Update product packaging design
2. Change in product formula
3. Company rebranding
4. Public relations crisis and recovery
5. Financial problems and budget constraints
6. New product packaging technology

#### The Task

An oatmeal company has recently seen an increase in the cost to produce their packaging container. They have hired you as a consultant. Your job is to evaluate the dimensions and shapes of a possible new container that will hold the most oatmeal while minimizing the cost associated with the packaging material and then model the different dimensions and shapes and build the best design. Each container can only be made with the amount of material equivalent to one poster board. "Your work must include:

- a. 2d representations of your 3d figures with defining characteristics labeled and corresponding surface areas and volumes calculated. You should have five representations: one prism, one pyramid, one cylinder, one cone, and one sphere. Try to maximize the volume for each of these shapes without using more than one posterboard's worth of material. (Play around with the [Shodor online interactive](#) or the [Learn Alberta interactive](#) to get a feel for how changing dimensions affects surface area and volume.)
- b. A [Tinkercad](#) representation of your prism, pyramid, cylinder, cone, and sphere.
- c. A physical model of your most effective design.
- d. An oral presentation that mathematically and verbally persuades the audience of your design."

2. Which of the six reasons you read about in the article explains why this company wants to change their packaging design?

### 5. Financial problems and budget constraints

3. What are the criteria and constraints of the task?

#### criteria

Maximize volume

Freestanding package

Can pour out oatmeal easily

#### constraints

minimize surface area

material must be equivalent

or less than one

poster board

4. On the activity sheet, draw a 2d representation of your optimal design for each shape with defining characteristics labeled. Then show all work for the surface area and volume calculation for each.

### Rectangular Prism

First of all, in order to do any of these shapes, we need to know how much surface area we can use. A standard poster board is 22 inches by 28 inches, or 55.9 cm x 71.1 cm. That gives us a total working area of about 3974.5 square cm.

Based on the [Shodor interactive](#), the closer a rectangular prism gets to a cube, the larger the volume gets compared to the change in surface area. In a cube, length = width = height. Therefore, we want  $l=w=h$ .

$$S.A. \text{ RECT. PRISM} = 2(wl + hl + hw)$$

$$3974.5 \text{ cm}^2 = 2(wl + hl + hw)$$

$$1987.25 \text{ cm}^2 = wl + hl + hw$$

$$1987.25 \text{ cm}^2 = ll + ll + ll$$

$$1987.25 \text{ cm}^2 = 3l^2$$

$$\sqrt{662.42 \text{ cm}^2} = \sqrt{l^2}$$

$$l \approx 25.7 \text{ cm}$$

$$V_{\text{RECT. PRISM}} = lwh$$

$$V_{\text{RECT. PRISM}} = (25.7)^3$$

$$V_{\text{RECT. PRISM}} \approx 16974.6 \text{ cm}^3$$

$$l \approx 25.7 \text{ cm}$$

$$w \approx 25.7 \text{ cm}$$

$$h \approx 25.7 \text{ cm}$$

$$\text{Actual S.A.} = 2(25.7^2 + 25.7^2 + 25.7^2)$$

$$\text{S.A.} = 3962.94 \text{ cm}^2$$



### Square Pyramid

After exploring surface area and volume pyramids, I noticed an interesting pattern between them. To a point, increasing height over the base dimensions yields a greater volume. After that critical point, it has the opposite effect. There is interesting calculus behind this, but it's outside the scope of this task. Accept anything from your students within reason as long as it's justified. For example, I would give full credit for either pyramid 1 or 2 because they are in the same

ballpark, but I would not give full credit for pyramid 3 since it's less than half the volume of the others. Clearly, there is a more effective design than pyramid 3.

PYR 1

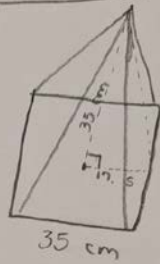
$$l = 35 \text{ cm } w = 35 \text{ cm } h = 35 \text{ cm } l = 39.1$$

$$l^2 = 35^2 + 17.5^2$$

$$l^2 = 1225 + 306.25$$

$$l^2 = 1531.25$$

$$l = 39.1$$



$$S.A. \text{ PYRAMID} = \frac{1}{2} lp + B$$

$$S.A. = \frac{1}{2} (39.1)(35 * 4) + (35 * 35)$$

$$S.A. = 2737 + 1225$$

$$S.A. = 3962 \text{ cm}^2$$

$$\text{Volume PYRAMID} = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (35 * 35)(35)$$

$$V = 1429.17 \text{ cm}^3$$

OR

PYR 2

$$l = 30 \text{ cm} \quad w = 30 \text{ cm} \quad h = 48.5 \text{ cm} \quad L = 50.7 \text{ cm}$$

$$15^2 + 48.5^2 = L^2$$

$$L = 50.7$$

$$S.A._{\text{PYR}} = \frac{1}{2} Lp + B$$

$$S.A. = \frac{1}{2} (50.7)(30 \times 4) + 30^2$$

$$S.A. = 3042 + 900$$

$$S.A. = 3942 \text{ cm}^2$$

$$V_{\text{PYR}} = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (30^2)(48.5)$$

$$V = 14550 \text{ cm}^3$$

★ A base edge of 31 and height of 46 were the best integers I found ☺

$$V_{\text{max}} \approx 14735.3 \text{ cm}^3$$

$$S.A. \approx 397056 \text{ cm}^2$$



PIR 3

$l = 10\text{cm}$   $w = 10\text{cm}$   $h = 193\text{cm}$   $l \approx 193.1\text{cm}$

$$5^2 + 193^2 = l^2$$
$$l = 193.1\text{cm}$$
$$S.A. = \frac{1}{2}lp + B$$
$$S.A. = \frac{1}{2}(193.1)(40) + (10^2)$$
$$S.A. = 3861.3 + 100$$
$$S.A. = 3961.3\text{ cm}^2$$
  
$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(10^2)(193)$$
$$V = 6433.3\text{ cm}^3$$

## Cylinder

Using the Learn Alberta Interactive, I noticed that volume was maximized when the height was about double the radius in the cylinder. Therefore  $h = 2r$ .

$$S.A. \text{ CYLINDER} = 2\pi r^2 + 2\pi r h$$

$$3974.5 \text{ cm}^2 = 2\pi r^2 + 2\pi r h$$

$$3974.5 \text{ cm}^2 = 2\pi r^2 + 2\pi r (2r)$$

$$3974.5 \text{ cm}^2 = 2\pi r^2 + 4\pi r^2$$

$$3974.5 \text{ cm}^2 = 6\pi r^2$$

$$662.42 \text{ cm}^2 = \pi r^2$$

$$210.85 \text{ cm}^2 = r^2$$

$$r \approx 14.5 \text{ cm}$$

$$h \approx 29 \text{ cm}$$

$$\text{Actual S.A.} = 2\pi r^2 + 2\pi r h$$

$$\text{Actual S.A.} = 2\pi(14.5)^2 + 2\pi(14.5)(29)$$

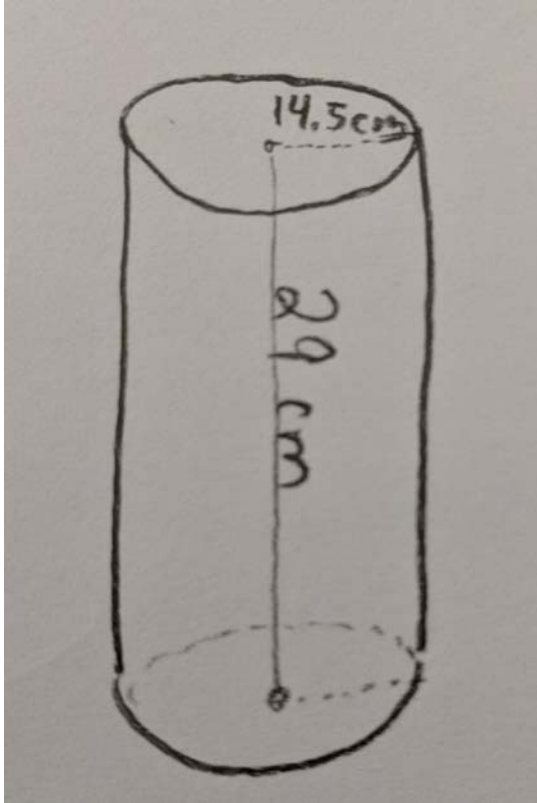
$$S.A. = 1321.04 + 2642.08$$

$$S.A. = 3963.1 \text{ cm}^2$$

$$V_{\text{CYLINDER}} = \pi r^2 h$$

$$V = \pi(14.5)^2(29)$$

$$V \approx 19155.1 \text{ cm}^3$$

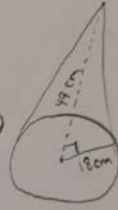


### Cone

The volume of a cone with respect to surface area follows a similar pattern as a pyramid. Again, accept all reasonable responses. For example:

Example

$$r = 18 \text{ cm} \quad h = 49 \text{ cm} \quad l \approx 52.2 \text{ cm}$$



$$S.A._{\text{cone}} = \pi r l + \pi r^2$$

$$S.A. = \pi(18)(52.2) + \pi(18)^2$$

$$S.A. = 2951.84 + 1017.88$$

$$S.A. \approx 3969.7 \text{ cm}^2$$

$$l = \sqrt{18^2 + 49^2}$$
$$l \approx 52.2 \text{ cm}$$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (18)^2 (49)$$

$$V \approx 16625.3 \text{ cm}^3$$

**Sphere**

SPHERE

$$S.A. \text{ SPHERE} = 4\pi r^2$$

$$3974.5 \text{ cm}^2 = 4\pi r^2$$

$$993.625 \text{ cm}^2 = \pi r^2$$

$$316.28 = r^2$$

$$r \approx 17.7 \text{ cm}$$

$$\text{Actual S.A.} = 4\pi (17.7)^2$$

$$S.A. = 4\pi (313.29)$$

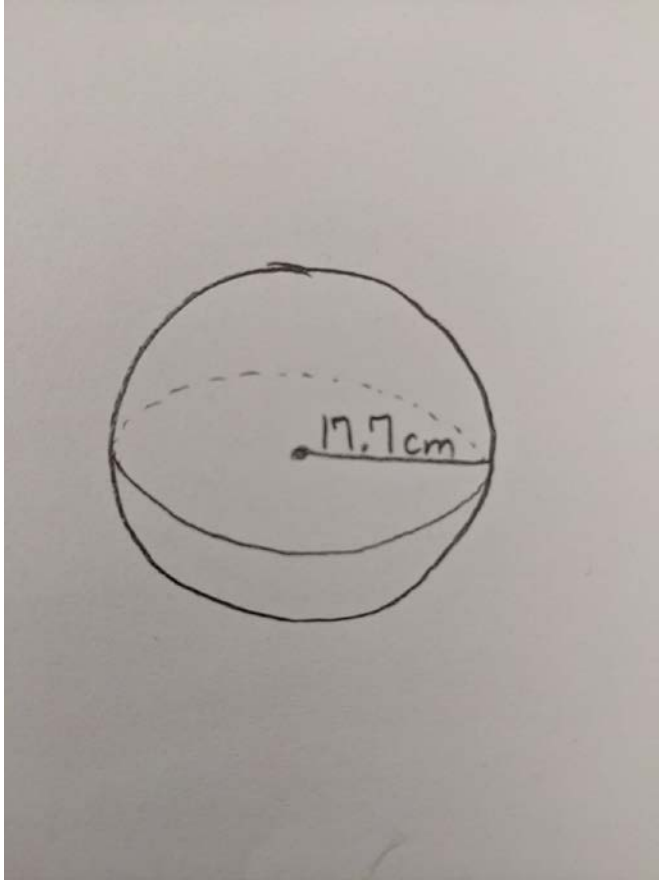
$$S.A. \approx 3936.9 \text{ cm}^2$$

$$\text{Volume}_{\text{SPHERE}} = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (17.7)^3$$

$$V = \frac{4}{3}\pi (5545.233)$$

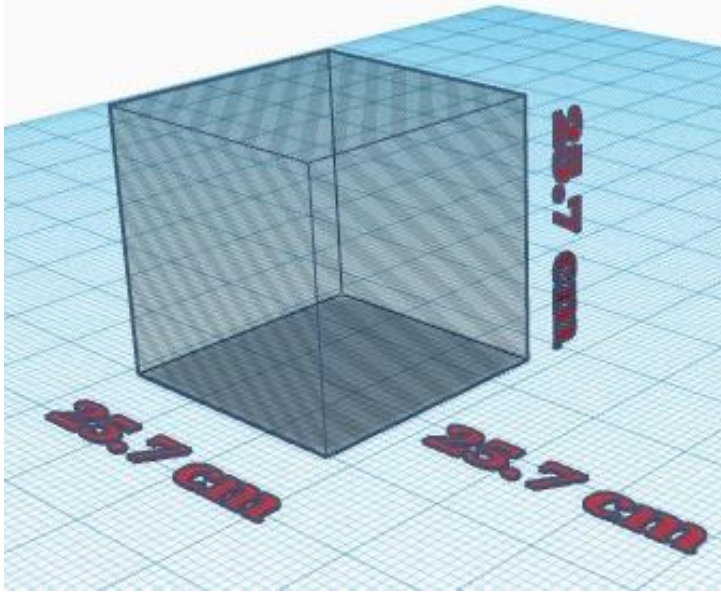
$$V \approx 23227.8 \text{ cm}^3$$



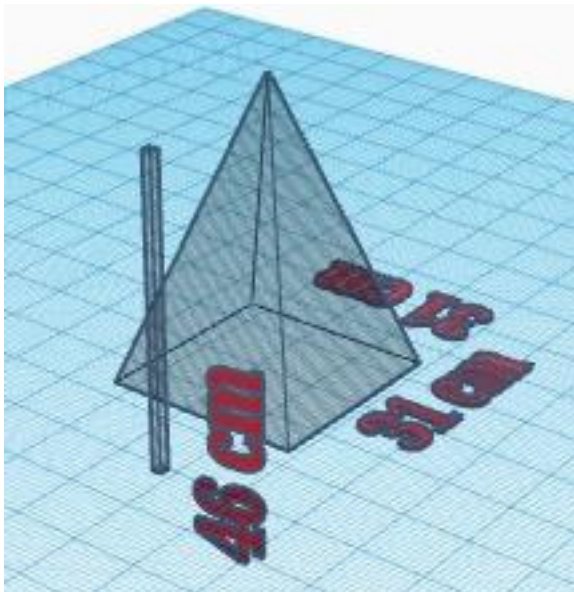
5. Now you will be using [Tinkercad](#) to make 2d models of your shapes. Each student needs to make at least one. When you are done, either export your shapes to me, or show me in person so I can initial beside each shape.

See the prism, pyramid, and cylinder I made for samples.

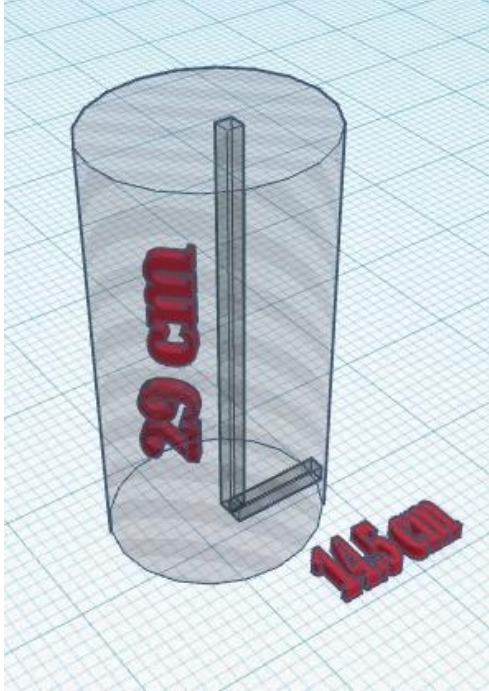
**Prism**



Pyramid



Cylinder



6. Physical Model - You will turn in your physical model with your activity sheets.

Answers will vary. Something to consider: Even though the sphere optimizes volume for a given surface area, it doesn't meet the other implicit criteria of being freestanding and easy to use. In my opinion, a cylinder with a radius of 14.5 cm and a height of 29 cm is the most effective design. Remember, though, the students do not have to find the absolute optimal design. They just have to be able to defend whatever design they chose with mathematical evidence and logic.

7. Write an outline for your defense articulation/pitch. You should include the elements of a good pitch as well as supporting mathematical evidence. Use [this Forbes site](#) for pitch elements. You will be scored on sound evidence and logic, proper vocabulary, command of the English language, appropriate verbal and nonverbal presentation skills, and tone appropriate to the audience, topic, and purpose.

Answers vary. Use the [rubric](#) for scoring guide.



8. Critique the reasoning of each group. Jot down notes of comments you want to make or questions you want to ask them.

Answers vary. Use the [rubric](#) for scoring guide.

## Designing Product Packaging Rubric

	0	1	2	3
Identify criteria and constraints of a design task.	No criteria or constraints have been identified.	Student identifies the criteria as holding the most oatmeal and constraint as minimizing cost.	Student meets the criteria for 1 plus relates that to volume and surface area.	Student meets the criteria for 2 plus identifies other more implicit constraints as well.
Brainstorm designs that meet criteria and constraints.	There is no audible or visual evidence of brainstorming.	There is evidence of brainstorming, but it was incomplete.	There is evidence of complete brainstorming.	Meets the criteria for 2 plus there is evidence that all members meaningfully contributed to the brainstorm.
Draw 2d representations of a prism with defining characteristics labeled.	No 2d representation is attempted.	A 2d representation is attempted, but it does not accurately reflect a prism.	A 2d representation that accurately reflects a prism is shown.	Meets the criteria for 2 plus length, width, and height are labeled.
Know the formula for the volume of a prism and correctly apply it to real world scenarios.	No attempt at volume calculation has been made.	The correct volume formula was identified but was incorrectly applied.	The correct volume is listed but no work or reference to the formula is shown.	The correct volume formula was identified and correctly applied.
Draw 2d representations of a pyramid with defining characteristics labeled.	No 2d representation is attempted.	A 2d representation is attempted, but it does not accurately reflect a pyramid.	A 2d representation that accurately reflects a pyramid is shown.	Meets the criteria for 2 plus length, width, height, and slant height are labeled.
Know the formula for the volume of a pyramid and correctly apply it to real world scenarios.	No attempt at volume calculation has been made.	The correct volume formula was identified but was incorrectly applied.	The correct volume is listed but no work or reference to the formula is shown.	The correct volume formula was identified and correctly applied.
Draw 2d representations of a cylinder with defining characteristics labeled.	No 2d representation is attempted.	A 2d representation is attempted, but it does not accurately reflect a cylinder.	A 2d representation that accurately reflects a cylinder is shown.	Meets the criteria for 2 plus radius or diameter and height are labeled.
Know the formula for the volume of a cylinder and correctly apply it to real world scenarios.	No attempt at volume calculation has been made.	The correct volume formula was identified but was incorrectly applied.	The correct volume is listed but no work or reference to the formula is shown.	The correct volume formula was identified and correctly applied.

Draw 2d representations of a cone with defining characteristics labeled.	No 2d representation is attempted.	A 2d representation is attempted, but it does not accurately reflect a cone.	A 2d representation that accurately reflects a cone is shown.	Meets the criteria for 2 plus radius or diameter, height, and slant height are labeled.
Know the formula for the volume of a cone and correctly apply it to real world scenarios.	No attempt at volume calculation has been made.	The correct volume formula was identified but was incorrectly applied.	The correct volume is listed but no work or reference to the formula is shown.	The correct volume formula was identified and correctly applied.
Draw 2d representations of a sphere with defining characteristics labeled.	No 2d representation is attempted.	A 2d representation is attempted, but it does not accurately reflect a sphere.	A 2d representation that accurately reflects a sphere is shown.	Meets the criteria for 2 plus length, width, height, and slant height are labeled.
Know the formula for the volume of a sphere and correctly apply it to real world scenarios.	No attempt at volume calculation has been made.	The correct volume formula was identified but was incorrectly applied.	The correct volume is listed but no work or reference to the formula is shown.	The correct volume formula was identified and correctly applied.
Use CAD software to model 3d figures.	Students did not attempt to use CAD software for modeling.	Students used CAD software to generate models but the dimensions are unrealistic for the project requirements.	Students used CAD software to generate models, and the dimensions are realistic for the project requirements for at least three shapes.	Students used CAD software to generate models, and all dimensions are realistic for the project requirements.
Evaluate competing design solutions using a systematic process to determine how they meet the criteria and constraints of the problem.	An ineffective design solution is chosen and there is no evidence of a systematic process of determination.	An effective design solution is chosen, but there is no evidence of a systematic process of determination.	An ineffective design solution is chosen, but there is evidence of a systematic process of determination that may have been flawed in the math or logic.	An effective design solution was chosen and there is evidence of a systematic determination process of math and logic.
Build a physical model of an optimal 3d design.	There is no physical model.	A physical model is attempted, but it does not resemble the optimal 3d design.	A physical model of the optimal 3d design is built, but the dimensions were not preserved from the 2d representation.	A physical model of the optimal design is built and the dimensions of the model have been appropriately measured with a metric ruler to match the 2d representation.
Articulate a mathematical defense using evidence and logic.	No attempt is made to articulate a mathematical defense.	An attempt is made to articulate a mathematical defense, but it contains significantly flawed or lacking evidence or logic.	An attempt is made to articulate a mathematical defense that contains occasional flawed or lacking evidence or logic.	A mathematical defense with sound evidence and logic is articulated.

Articulate a mathematical defense using proper vocabulary.	There is no evidence of grade-appropriate vocabulary pertaining to English, science, technology, engineering, or math.	There is evidence of grade-appropriate vocabulary from at least one two subjects.	There is evidence of grade-appropriate vocabulary from at least three subject.	There is evidence of grade-appropriate vocabulary from at least four subjects.
Articulate a mathematical defense using command of the English language.	No attempt is made to use proper grammar or formal language in the defense.	Improper grammar and informal language are frequently used in the defense.	Improper grammar and informal language are only sometimes used in the defense.	Proper grammar and academic language rather than informal language are almost exclusively used in the mathematical defense.
Articulate a mathematical defense using appropriate verbal and nonverbal presentation skills.	The defense is read. No presentation is attempted.	At least two of the following are appropriately used in the defense: body movement, eye contact, facial expressions, posture, volume, tone, and speed of speech.	At least four of the following are appropriately used in the defense: body movement, eye contact, facial expressions, posture, volume, tone, and speed of speech.	All of the following are appropriately used in the defense: body movement, eye contact, facial expressions, posture, volume, tone, and speed of speech.
Respond to audience questions and comments.	No attempt is made to respond to audience questions and comments.	An attempt was made to respond, but responses did not sufficiently answer the questions or address the comments.	Logically coherent responses were given but not within a suitable time frame or without appropriate presentation skills.	Logically coherent responses were given with appropriate presentation skills.
Critique the arguments of others.	Student did not engage in a critique either in writing or speech.	Student critiqued the arguments of others in writing or speech but the logic of the critiques were flawed.	Student thoughtfully and logically critiqued the arguments of others in writing or speech but not both.	Student thoughtfully and logically critiqued the arguments of others in writing and speech.
Work on a team respectfully and productively.	Members treated each other disrespectfully and did not often meaningfully contribute to the project.	Members were only meaningfully contributing sometimes during the project and did not always treat each other with respect.	Every member treated each other respectfully, but there were times when members were not meaningfully contributing. There is some evidence of goal setting and tracking and partitioning of duties.	Every member meaningfully contributed during the entire length of the project and treated each other respectfully. There is ample evidence of goal setting and tracking and partitioning of duties.
<b>COLUMN TOTALS:</b>				
<b>FINAL SCORE:</b>				

# 6 Reasons to Consider a Product Packaging Redesign

printrunner.com/blog/product-packaging-redesign/

By  
Kevin

January 4, 2017



The most important aspects of your marketing is arguably your product packaging. Especially for businesses with physical products, your packaging design can be the deciding factor on whether your item is purchased or not. In today's design-focused environment, customers place equal importance on the quality of your packaging design to the functional aspect of your product. If you want to stand out in the product aisle, you need to invest on your product packaging.

But when is the right time to consider a product packaging redesign?

The answer to the question varies across different industries. However, we can at least cover common indicators. Here are 6 reasons to consider a product packaging redesign.

## 1. Outdated Product Packaging Design.

Design trends come and go. For instance, the 90s-favored skeuomorphic designs, which refers to the design concept of making items resemble their real-world counterparts. This can be seen in computer interfaces and is primarily the reason why the save icon is a diskette. Most of the product packaging designs during this decade feature bright colors and a lot of design elements.

Today's design standards on the other hand tend to go the minimalist route. Designers put a lot of emphasis on white space, that is the blank space around the text and symbols. There is also a significantly lower amount of design elements.

Consider when your product design was created. Compare its design to the current trends in your industry's product packaging. If it's old, it definitely needs an upgrade.

## 2. Change in Product Formula

Products change over time. As new ingredients become available, your product formula will change in order to adapt. New technologies can change your product's functionalities and specifications. Your product may even require a new size or a new shape. These improvements can pile up and may significantly change how your customers perceive your product and your company.

To reflect these changes, you also need to change your product packaging design. Aside from the obvious need to change ingredients or change the packaging shape and size, it is also a good opportunity to advertise the improvements on your product formula g.

## 3. Company Rebranding

It's a trend in business lately to rebrand from the ground up. It's usually the legacy brands which have been around for decades who change to a more contemporary branding to reflect the modern times. Others change their logos and other company visuals to reflect a change in direction.

Whatever your reason may be, don't forget to update your packaging. It's important to make the design consistent with your own brand's new aesthetic.



#### 4. Public Relations Crisis and Recovery

Public relations nightmares can happen even to the best of us. Especially today when customer interaction happens almost immediately through social media, one bad piece of publicity could be fatal to your brand. It may take a while to recover from the effect of this negative publicity.

Take the time to consider modifying your product packaging for two reasons. One, it would show that you are moving on from the bad publicity you receive. And two, you are actively trying to improve your product and image, starting with the design.

#### 5. Financial Problems and Budget Constraints

Every business experiences financial problems. There may be some expenses that eat through your revenue. Furthermore, some product packaging can be costly. You may be too financially invested in your product packaging design without any noticeable return of investment.

In any case, consider redesigning your product packaging if you want to cut costs. Even a small savings per piece can add up.

#### 6. New Product Packaging Technology

New technology can result in a more efficient product packaging process. Furthermore, there may be new ways to design it that you are missing out on.

For instance, there are new styles to choose from when making [custom boxes](#) that may be a better fit for your products. You can even introduce visual improvements to your packaging. These include embossing or debossing an element to emphasize it, spot UV that highlights a portion of your design, or foil stamping to give a luxurious appeal. Don't miss out on new product packaging technologies. Incorporate them in your design to create a more powerful packaging redesign.

**When do you update your product packaging? Tell us your thoughts below.**

Recommended for You



Update your branding by using new technologies on product packaging.