# Introduction to Transformations

**Submitted by: Maria Rhodes, Geometry**
**Chattanooga Christian School, Chattanooga, TN**

**Target Grade**: Geometry

**Time Required**: 75 minutes

**Standards**

*Common Core Math Standards*

- **G.CO.A.2** (IFD) Represent transformations in the plane in multiple ways, including technology. Describe transformations as functions that take points in the plane (preimage) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).
- **G.CO.A.4** (IFD) Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**Lesson Objectives**

Students will:

- Represent translations using a graph, a table, and arrow notation.
- Describe the difference between rigid and non-rigid transformations.

**Central Focus**

This lesson illustrates a real-world connection to the students' lives. Video games are designed using mathematical transformations. In addition, they will complete problems involving the movement of a marching band across a field. Many of the students participate in band or ROTC, both of which require marching movements.

Key Terms: transformation, translation, reflection, rotation, dilation, preimage, image, vector

**Background Information**

This lesson is the third part to a three part lesson. Look for the following on the ORISE website:

- Lesson 1: Reflecting on Reflections
- Lesson 2: Rotations – All Turned Around

This lesson uses the students' prior knowledge of graphing on the coordinate plane and using multiple representations (tables, graphs, equations) to describe a function. This lesson also builds off students' prior learning as it requires them to graph figures on the coordinate plane, read graphs and tables, and write expressions in function notation. In order to do all of these tasks, the students must know coordinate notation and how to plot points on the coordinate plane. They must also use their knowledge of functions as having an input and an output. Because the students have taken and passed Algebra 1, they are familiar with linear functions and graphing points and lines on the coordinate plane. In addition, the students have used these skills in this course when they learned how to use the distance formula to calculate the distance between two points of the coordinate plane. This knowledge of functions and coordinate geometry will be activated during the warmup of this lesson and used to introduce transformations as functions that can be represented with the same mathematical tools used to represent linear functions.

Prior to this lesson, teachers should be familiar with the following terms: transformation, translation, reflection, rotation, dilation, preimage, image, and vector.

Transformation

- Changing a shape using turn, flip, slide, or resize.
  (Transformations (mathsisfun.com))

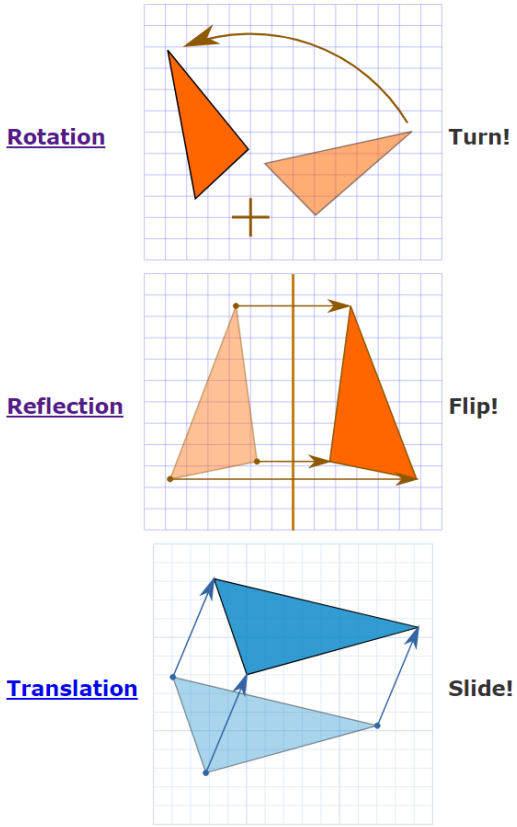



*Figure 1: https://www.mathsisfun.com/geometry/transformations.html*

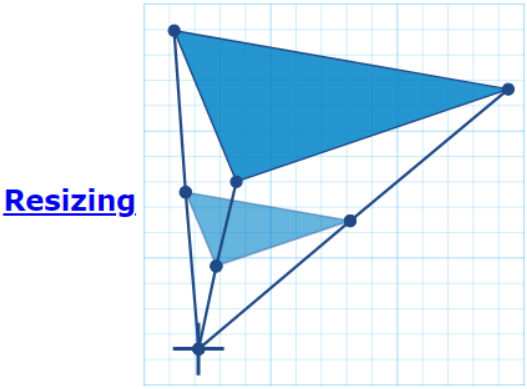



*Figure 2: https://www.mathsisfun.com/geometry/transformations.html*

Translation

- In Geometry, “translation” simply means moving without rotating, resizing, or anything else, just moving.
- To translate a shape, every point of the shape must move:
    - The same distance.
    - In the same direction.

  (Geometry Translation (mathsisfun.com))

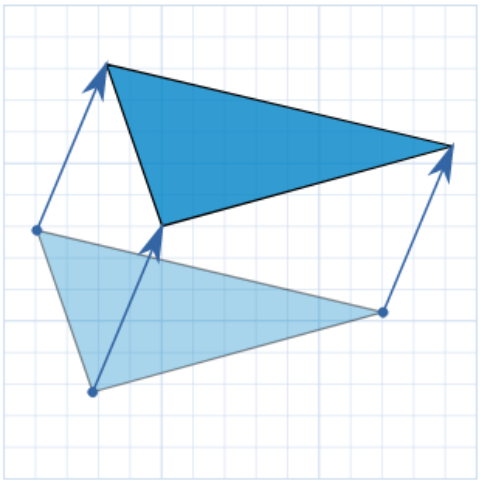


*Figure 3: https://www.mathsisfun.com/geometry/translation.html*

Reflection

- A reflection is a flip over a line.
- Every point is the same distance from the central line and the reflection has the same size as the original image.

  (Geometry - Reflection (mathsisfun.com))

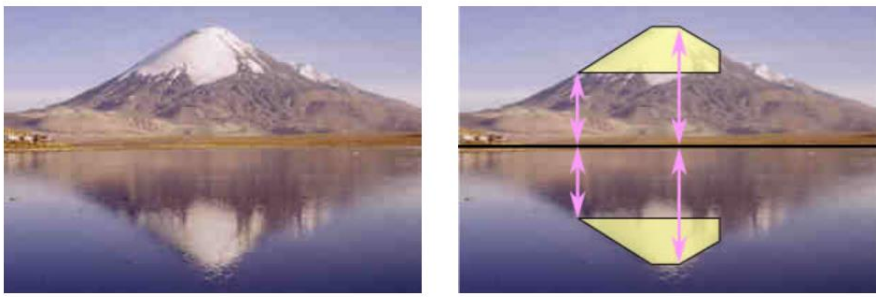


*Figure 4: https://www.mathsisfun.com/geometry/reflection.html*

Rotation

- Rotation means turning around a center.
- The distance from the center to any point on the shape stays the same.
- Every point makes a circle around the center.
  (Geometry Rotation (mathsisfun.com))

*Figure 5: https://www.mathsisfun.com/geometry/rotation.html*

Dilation

- To resize something.
  (Dilation Definition (Illustrated Mathematics Dictionary) (mathsisfun.com))

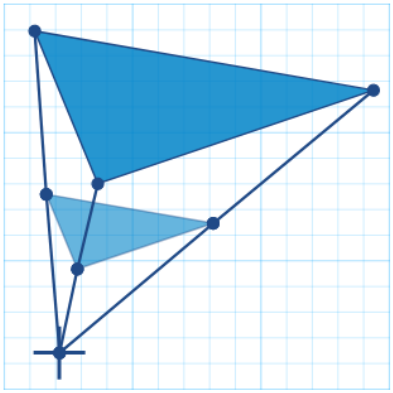


*Figure 6: https://www.mathsisfun.com/definitions/dilation.html*

Preimage

- The original, unaltered shapes are called preimages.
  (Preimage & Image - Calculus How To)

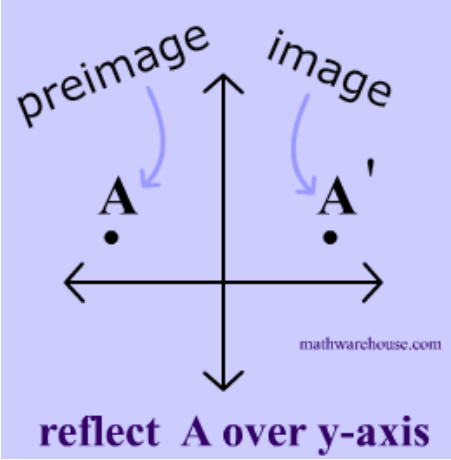



*Figure 7: https://www.mathwarehouse.com/transformations/*

Image

- The new (transformed) shapes are called images.
  (Preimage & Image - Calculus How To)

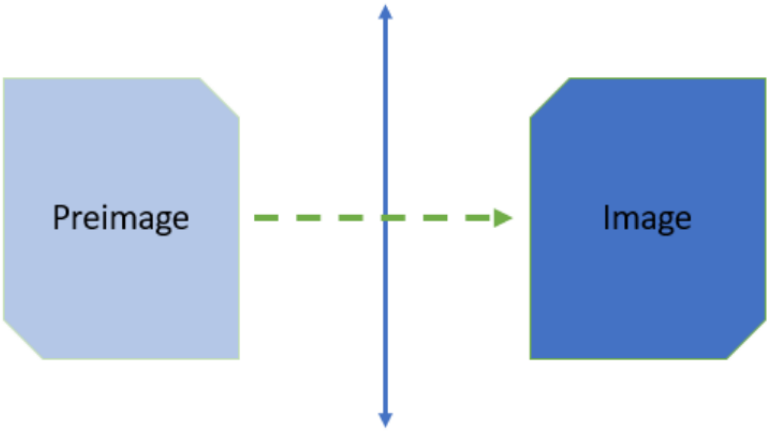



*Figure 8: https://www.calculushowto.com/preimage-image/*

Vector

- A vector has magnitude (size) and direction. (Vectors (mathsisfun.com))

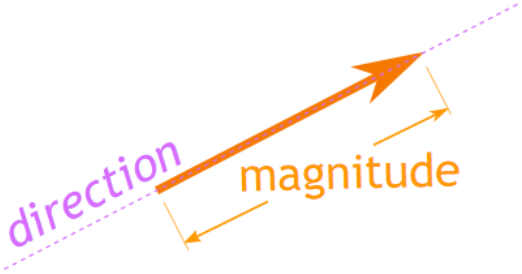



*Figure 9: https://www.mathsisfun.com/algebra/vectors.html*

**Materials**

- TV or projector
- Whiteboard
- Computer
- Paper
- Translation worksheet
- Translation exit ticket
- Transformations Intro slide
- Transformations Coordinate Rules Worksheet (optional)
- Menti Code (create at: Interactive presentation software - Mentimeter)
    - Mentimeter is an interactive presentation software where the audience can input words or ideas that will then be collected to form a word cloud.
- Youtube video: Super Mario Bros - Full Game Walkthrough https://www.youtube.com/watch?v=cWOkHQXw0JQ
    - This video shows the screen of an old Super Mario Bros game as someone plays it. All of the movements that occur on the screen are transformations.

*Figure 10: https://www.mariogolf.net/mario-games-that-are-easy/*

**Instruction**

Introduction (10 minutes)

- Instruct students to complete a warmup in which they brainstorm everything they remember about functions.
    - Have students submit their reasoning and explanations in Menti, which creates a word cloud.

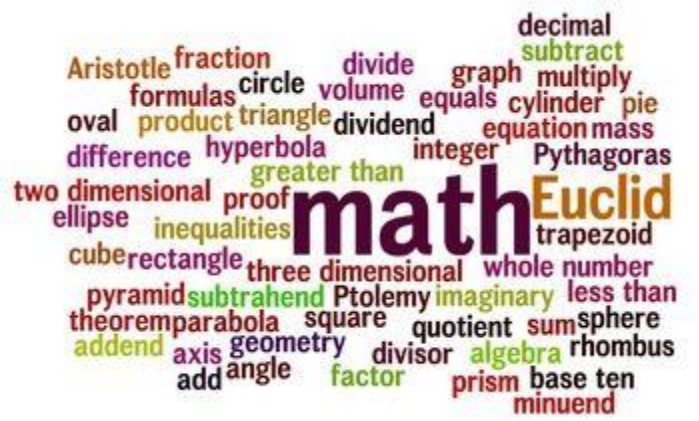



*Figure 11: https://www.pinterest.com/pin/448811919086158565/*

- Discuss the words and phrases in the word cloud that the students created.
- Prompt the students to discuss: "Where have you used functions before? Are equations functions?"
- Once the class recognizes linear equations as functions, discuss the question, "Are there other ways to represent linear functions?"
    - This question will lead to a discussion about different ways to represent a function, including a table, an equation, and a graph.
- Here are some questions to ask the students during the introduction of the lesson:
    - How would you define the word function?
    - What is a function outside of mathematics?
    - Is the equation of a line a function?
    - What are different ways you used to represent a linear function in Algebra I?

Activity (45 minutes)

Part One (20 minutes)

- Hand out paper for students to create a table and write what they observe in the video.
- Show the students the first 3 minutes of a walkthrough of Mario Super Bros 1985: https://www.youtube.com/watch?v=cWOkHQXw0JQ


*Figure 12: https://www.youtube.com/watch?v=lE_XYVdFUBA*

- While the students watch the video, tell them write down descriptive words for the movements they see in the video.
- Have a whole class discussion where students share their words with the class. Example words might include “slide,” “flip,” “turn,” and “grow.”
- Tell the students that these are all transformations and will learn the mathematical terms for these movements: translation, reflection, rotation, dilation.
- Instruct the students to fill out a table to organize their learning of these new vocabulary words.
- Have the students discuss the properties of these functions by answering the question, “What do you think is the input and output of transformations?”
- Give students time to discuss their ideas with their partners and have the students share their ideas in a whole class discussion.
- Introduce the vocabulary “image” and “preimage” as precise descriptions for the input and output of a transformation.
- Discuss the difference between rigid and non-rigid transformations, have the students discuss the question, “How are dilations different from the other three transformations?”
- Optional: Have the students fill out the transformations coordinate rules, specifically the translation section.

- Here are some questions to ask during this part of the activity:
    - What words describe the movement of the images in this video?
    - What are the inputs and outputs of these functions?
    - How are dilations different from the other transformations?
    - What is happening to each coordinate during the translation?
    - How do the coordinates of each point change?
    - How could you use ordered pairs to express this change?

Part Two (25 minutes)

- Hand out the translations worksheet.
- Have the students work in groups of 3-4 to complete problems 1-6 on the worksheet.
    - These problems require students to describe the movement of a point as it undergoes a translation, explain why someone's conjecture is wrong, and draw the image of a translation when given its function notation.
- When the groups have completed these problems, reconvene to discuss the answers.
- If some groups finish early during group work, direct them to try problems 7-8, which explore how to describe a translation off of the coordinate plane.
- Here are some questions to ask during this part of the activity (the numbers refer to the problem):
    - 1-3. What do a and b represent in arrow notation?
    - 4. How much did the X move horizontally and vertically? What does that mean about the rest of the shape?
    - 5. What error has the friend made? What should the new coordinates be?
    - 6. How is the shape moving? Does it move horizontally, vertically, or both? Can you describe out loud how this shape moves? What vector describes the movement of this object?
    - 7-8. What is the direction and magnitude of line segment EB?

Closure (15 minutes)

- As the groups wrap up their work, enter into a whole-group discussion about what was the most difficult concept the students encountered during the lesson.
- Hand out the exit tickets to the students to complete before they leave.
    - The first question on the exit ticket will require the students to describe the difference between rigid and non-rigid transformations.
    - The next question gives a graph showing the pre-image and image of a translation. The problem asks the students to write a function that describes the translation and to give a written description of how the figure moved during the translation.

**Differentiation**

Grouping:

- Students will be grouped by the arrangement of their desks into groups of 3-4 students.
- These groups will be mixed ability grouping for peer support and collaborative learning.
- The peer interaction will act as a means of differentiation. The heterogeneous groups will allow for more advanced students to solidify their understanding as they support their peers, and students who need extra support will have guidance from their group members.

Student who struggles with algebra:

- The use of visuals will help support this student as they read graphs and draw figures.
- The teacher can make sure to check this student's work to ensure that they are using the proper coordinates.
- Write on the board a guide about reading function notation that they can refer to. The guide will say "(horizontal input, vertical input) → (horizontal output, vertical output)".

Student who processes information slowly:

- Heterogeneous grouping will provide peer support as the student thinks through the problems.
- Instruct this group to first discuss what they think the problems are asking for before beginning to work. This initial discussion will help this student process what the question is asking.
- The students will be supported by the range of activities, as they will only be expected to finish problems 1-6 and not 7-8.
- The class discussion after the group work will help catch this student up to the others.

Students who need an extra challenge:

- The group work is a tiered assignment. All of the students are expected to complete problems 1-6, but only some will complete problems 7-8.
- Having extra problems accommodates students who work at a faster pace and process information more quickly. Problems 7-8 deal with translations off the coordinate plane and have more complex notation.
- Students who finish 7-8 will be asked to present their work to the class during the final discussion.
- For students who have additional time after completing all problems (1-8), give them the choice of researching either the similarities or the differences in geometric transformations and linear function transformations or how vectors are used in higher-level mathematics.

**Assessment**

*Formative Assessment*

- During the class discussions, the teacher will be able to monitor students' understanding as they answer questions. The teacher will call on both volunteers and non-volunteers to get a good sample of the understanding of all the students.
- During group work, the teacher will walk around the room to listen to students' discussion and check their work. The teacher can see if they are on the right track, if they are stuck, or if they are working with a misconception. The teacher will use the questions mentioned above to help the teacher assess their understanding of the material as well as the work they produce. The information obtained during the group work will be used to guide the whole group discussion to address any misconceptions and challenge areas.

*Summative Assessment*

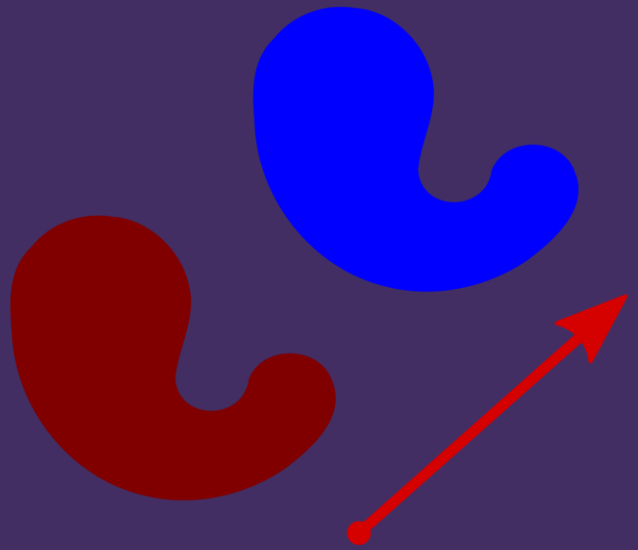
- The exit ticket assesses the students' understanding of the difference between rigid and non-rigid motions. It also assesses their ability of reading graphs and writing coordinates in the correct position with coordinate notation. Their reasoning is assessed as they connect the graphical representation of the translation to the algebraic function notation.

# Transformations


Write down all the words you can think of that describe the movement of the objects on the screen.

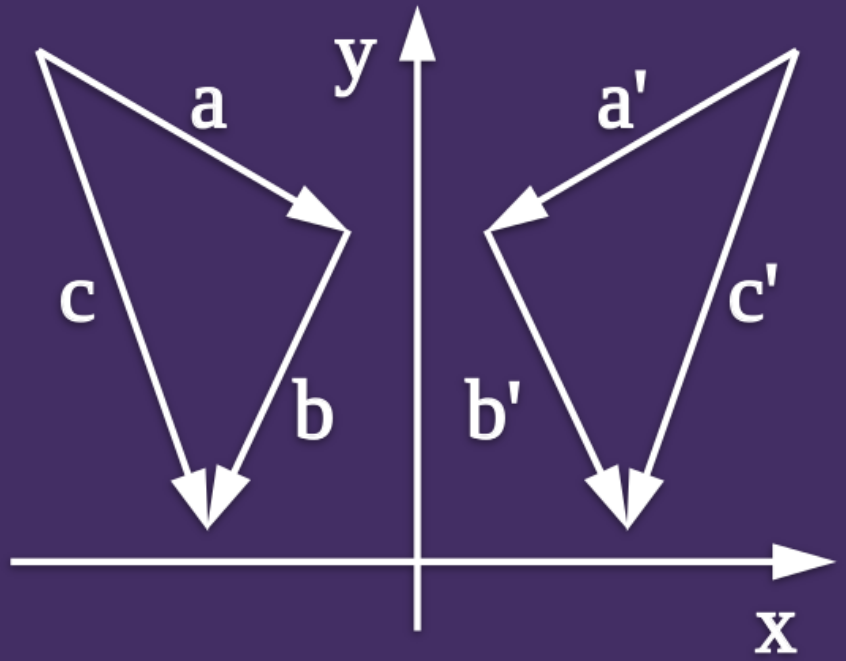
# Translations

"slide"



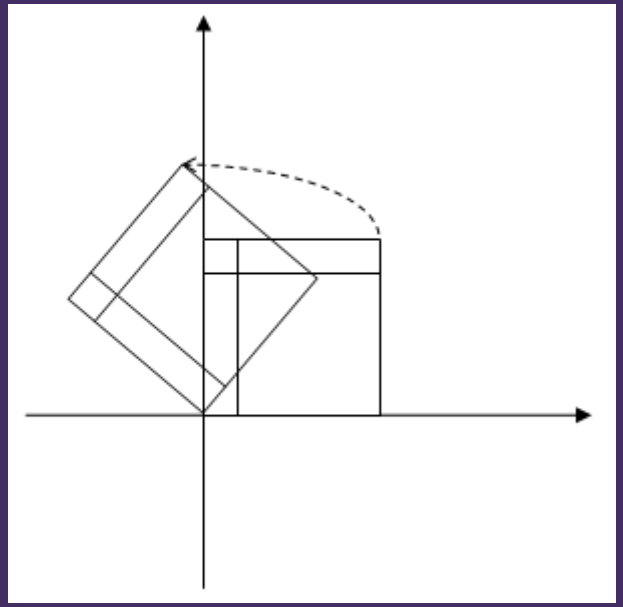
# Reflections

"flip"



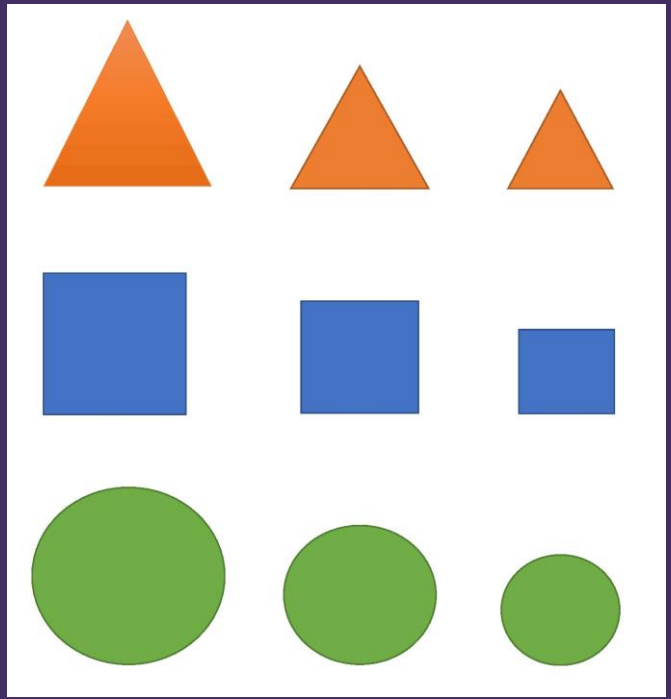
# Rotations

"turn"



# Dilations

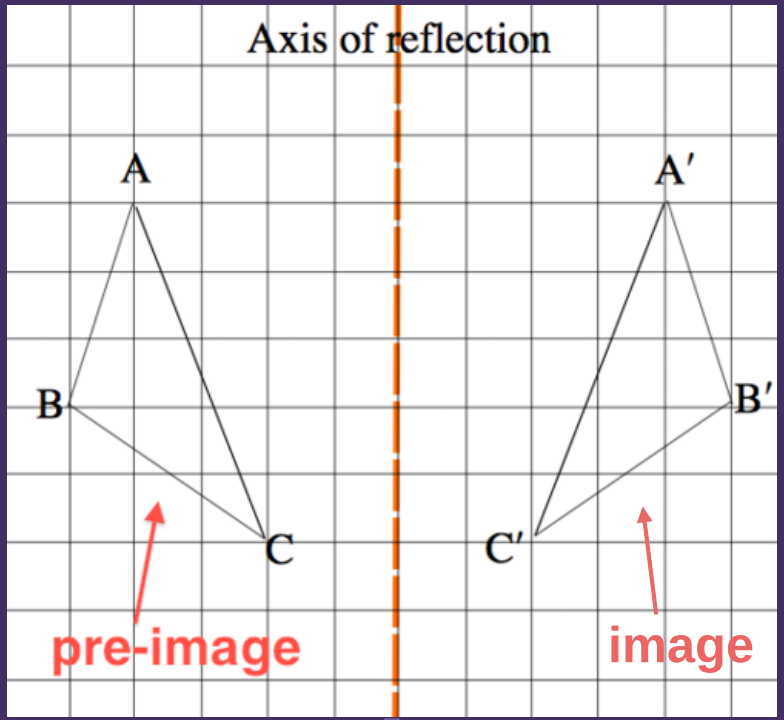
"grow/shrink"



*nonrigid transformation: it changes the size of the shape!

# Transformations are Functions!

Input: pre-image

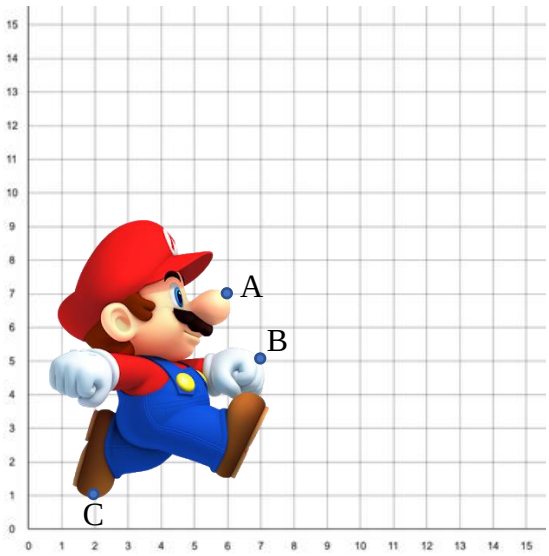


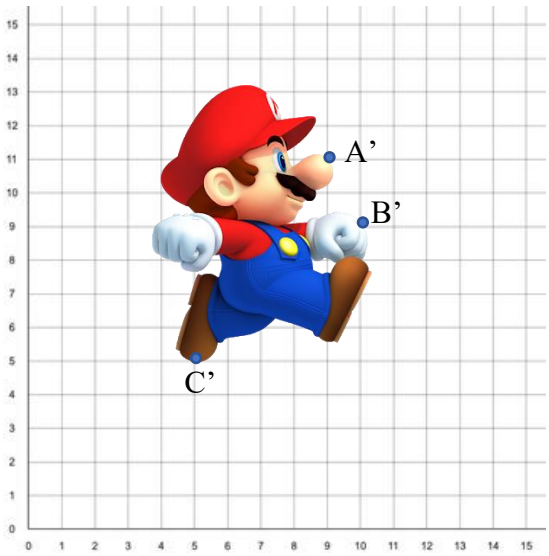

Output: image

We always denote the image with the prime (‘) symbol

# A Move in the Right Direction

## Translations in Action



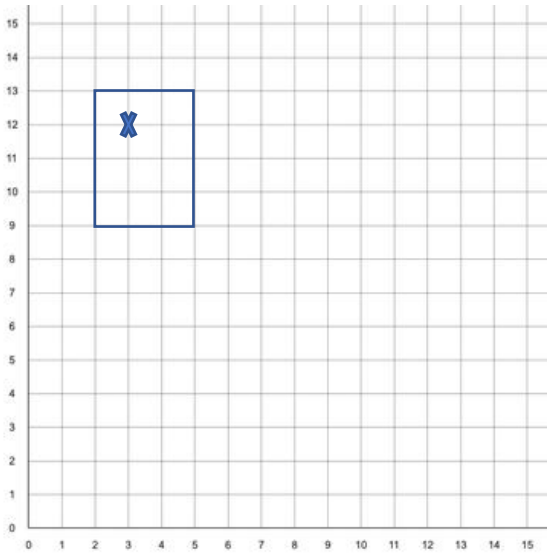

The graphs above show Mario before and after a translation. A translation causes the figure to slide up, down, left, right, or diagonally. The distance and direction of a translation is called a direction line segment (or vector).

1. Draw the directed line segment on the first graph that shows that path of A to A'.

2. Complete the table to show how the translation affects the points shown.

| Point | Pre-image | Image |
|---|---|---|
| A | (6, 7) | |
| B | (7, 5) | |
| C | | |

3. On the coordinate plane, a translation can be described using the following arrow notation: $(x,y) \rightarrow (x + a, y + b)$, in which a and b are positive or negative constant. Based on the table, what is the arrow notation for this translation?



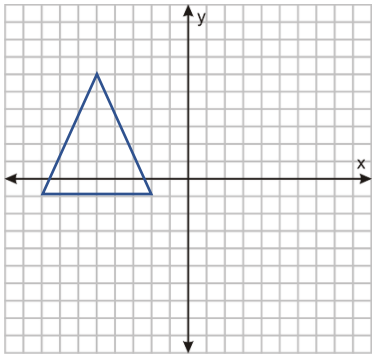
When undergoing a translation, the X moves from (3, 12) to (10, 4).

4. Draw the rectangle after the translation.

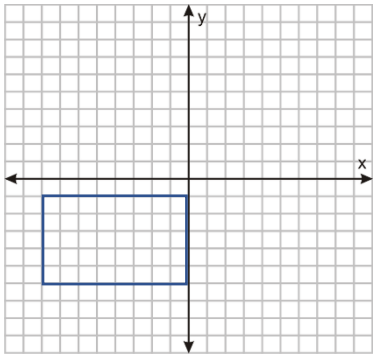
5. Josie states that the corner point at (2, 9) is now at (16, -6). What did Josie do wrong to come to this incorrect conclusion?

6. Draw the image of the figure described by the arrow notation.

a. (x, y) → (x + 2, y - 3)



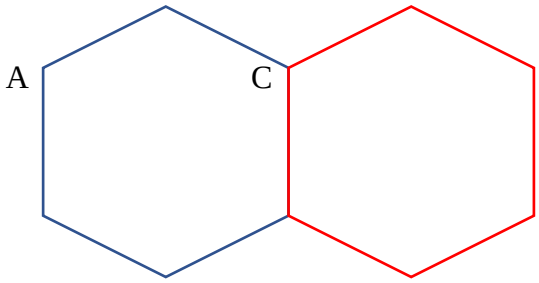

b. (x, y) → (x, y + 5)



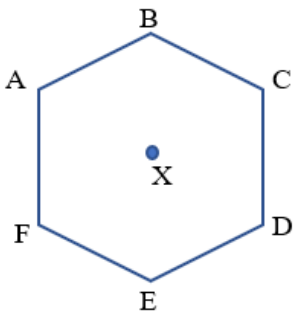
## Extension :

We can perform translations off the coordinate plane, and we still describe these translations using directed line segments.

For instance, the red shape is the image of the blue shape under the translation described by the directed line segment $\overline{AC}$.



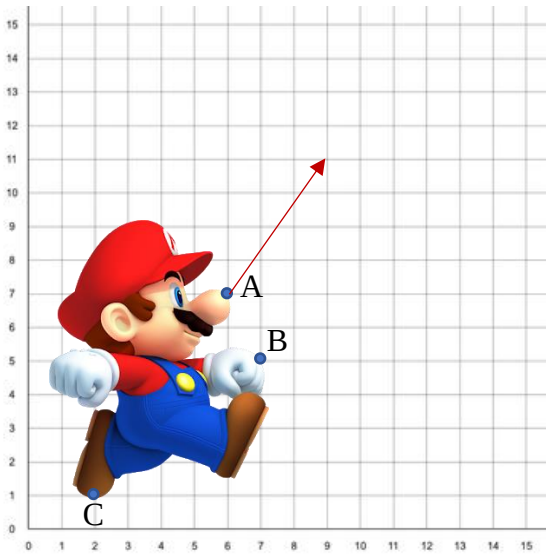

7. Draw the image of the translation described by the directed line segment $\overline{EB}$. Label the points of the image using the appropriate symbols.

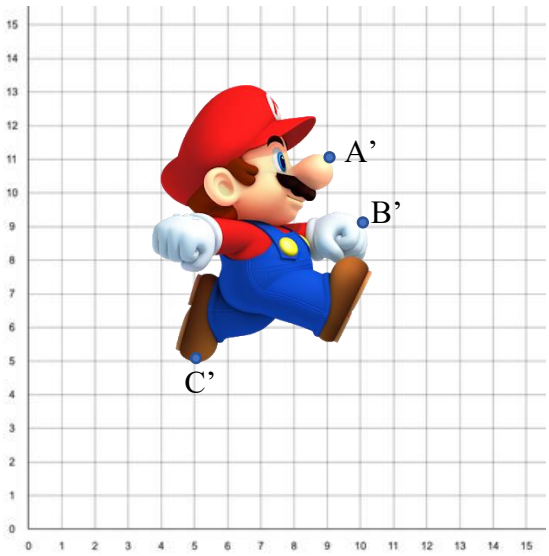



8. Which point would map onto X if you translated the figure with the directed line segment $\overline{XB}$?

# A Move in the Right Direction

## Translations in Action

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Answer shown in the graph above.

2. Complete the table to show how the translation affects the points shown.

| Point | Pre-image | Image |
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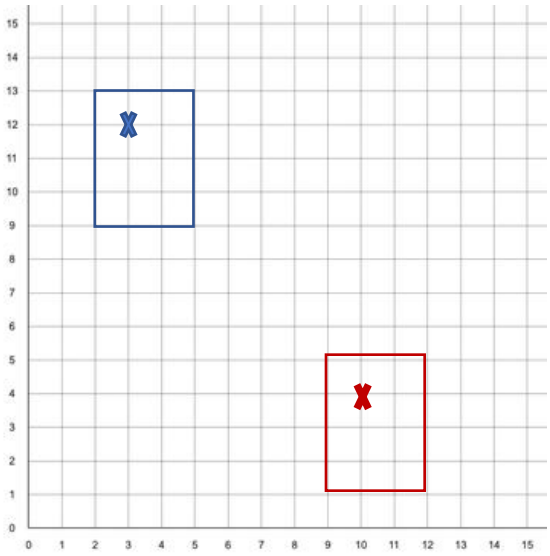
$(x,y) \rightarrow (x + 3, y + 4)$



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4. Draw the rectangle after the translation.

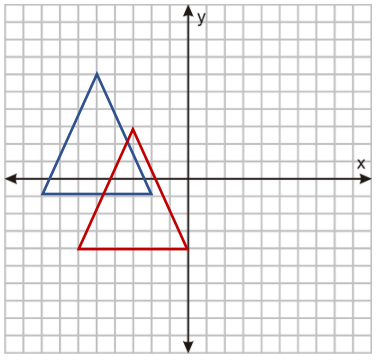
Drawn in red.

5. Josie states that the corner point at (2, 9) is now at (16, -6). What did Josie do wrong to come to this incorrect conclusion?

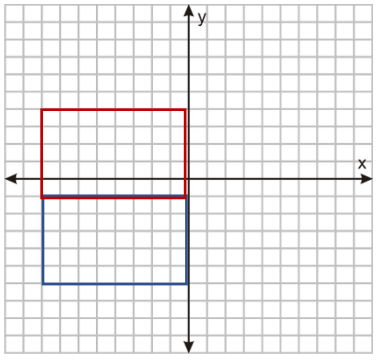
Answers may vary.

6. Draw the image of the figure described by the arrow notation.

a. (x, y) → (x + 2, y - 3)




b. (x, y) → (x, y + 5)



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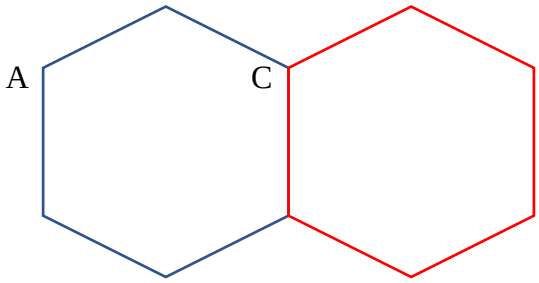
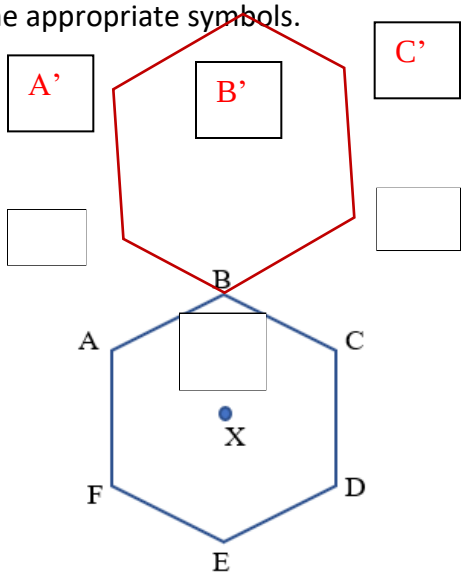



7. Draw the image of the translation described by the directed line segment $\overline{EB}$. Label the points of the image using the appropriate symbols.




8. Which point would map onto X if you translated the figure with the directed line segment $\overline{XB}$?

B

# Exit Ticket

Name: _____

1. Describe the difference between a rigid and nonrigid transformation.

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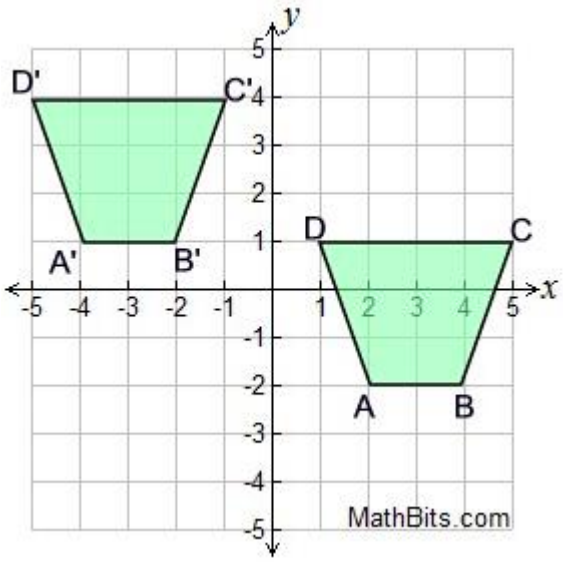


2. Identify the type of transformation above. Represent the transformation as a function using arrow notation.

3. Write a description of the movement of the figure above in your own words.

# Exit Ticket

Name: _____

1. Describe the difference between a rigid and nonrigid transformation.

Answers may vary.

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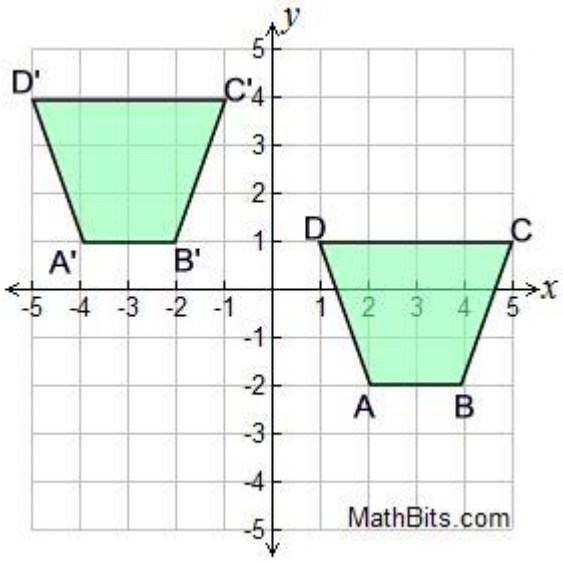


2. Identify the type of transformation above. Represent the transformation as a function using arrow notation.

Translation; $(x, y) \rightarrow (x + (-6), y + 3)$

3. Write a description of the movement of the figure above in your own words.

Answers may vary.

# Transformation Coordinate Rules

## Translations

## Reflections

Over the x-axis:

Over the y-axis:

Over the line y = x:

Over the line y = -x:

## Rotations

Center of rotation (0,0)

90˚ counterclockwise or ____ clockwise turn:

180˚ counterclockwise or ____ clockwise turn:

270˚ counterclockwise or ____ clockwise turn:

# Transformation Coordinate Rules KEY

## Translations

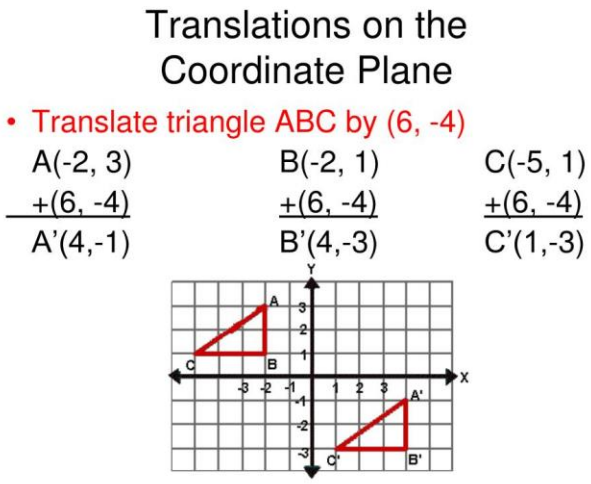


*Figure 1: https://www.slideserve.com/theola/translations-on-the-coordinate-plane*

## Reflections

Over the x-axis: (x, y) → (x, -y)

Over the y-axis: (x, y) → (-x, y)

Over the line y = x: (x, y) → (y, x)

Over the line y = -x: (x, y) → (-y, -x)

# Rotations

Center of rotation (0,0)

90˚ counterclockwise or 270 degrees clockwise turn:

$(x,y) \rightarrow (-y, x)$

180˚ counterclockwise or 180 degrees clockwise turn:

$(x,y) \rightarrow (-x, -y)$

270˚ counterclockwise or 90 degrees clockwise turn:

$(x,y) \rightarrow (y, -x)$